SENSITIVITY ANALYSIS OF A RELIABILITY ESTIMATION PROCEDURE FOR A COMPONENT WHOSE FAILURE DENSITY IS A MIXTURE OF EXPONENTIAL FAILURE DENSITIES

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The failure rate Λ, of components with exponential failure times, because of the manufacturing process of the components, behaves as a random variable. Assuming Λ is Gamma distributed with shape parameter α and scale parameter β, the reliability estimate of the components can be expressed as

(20. ABSTRACT Continued)

$$\hat{R}(t) = \frac{1}{(1 + \hat{\beta}t)^{\hat{\alpha}}}$$

where $\hat{\alpha}$ and $\hat{\beta}$ are the maximum likelihood estimators of α and β respectively. The accuracy of this estimate is analyzed when the distribution of Λ is discretely distributed rather than gamma distributed.

Sensitivity Analysis of a Reliability Estimation Procedure for a Component Whose Failure Density is a Mixture of Exponential Failure Densities

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I. INTRODUCTION

Let T be the life time of a component that is exponentially distributed with parameter Λ. Because of the manufacturing process and different logistical forces acting on the components, Λ behaves as a random variable with a certain distribution G. The distribution of the time to failure, under the above assumptions, is called a mixed exponential distribution.

Let N components be put on test until a test time T_o is reached. Suppose that as a result of testing, $t_1, t_2, \ldots, t_k$ were life times of components that had failed and $t_{k+1}, t_{k+2}, \ldots, t_N$ were life times of components that had not failed. If G is a Gamma distribution with shape parameter α and scale parameter β, then α and β can be estimated by using the method of maximum likelihood.

The M.L.E. of the reliability function of the components can be expressed as

$$\hat{R}(t) = \frac{1}{(1 + \hat{\beta} t)^{\hat{\alpha}}}$$

where $\hat{\alpha}$ is the M.L.E. of α

$\hat{\beta}$ is the M.L.E. of β

Computer simulations were conducted to determine the sensitivity of this estimation procedure when the exponential distributions are mixed differently from the assumed Gamma distribution mixture.

II. STATEMENT OF THE MODEL

The model and the derivations in this section are discussed in detail in a paper by Myhre and Saunders [5].

Let T the life time of a component be exponentially distributed with parameter Λ, where Λ is a random variable with Gamma distribution G with shape parameter α and scale parameter β.

The density function of Λ is

$$g(\lambda) = dG(\lambda) = \frac{\lambda^{\alpha-1} e^{-\lambda/\beta}}{\Gamma(\alpha)\beta^{\alpha}} \tag{1}$$

The reliability of the component is

$$R(t) = P(T>t)$$

$$= \int_0^\infty P(T>t|\Lambda=\lambda)\ dG(\lambda)$$

$$= \int_0^\infty e^{-\lambda t} \frac{\lambda^{\alpha-1} e^{-\lambda/\beta}}{\Gamma(\alpha)\beta^{\alpha}} d\lambda$$

$$= \frac{1}{(1+\beta t)^{\alpha}} \int_0^\infty \frac{e^{-\lambda(\frac{1+\beta t}{\beta})} \lambda^{\alpha-1}}{\Gamma(\alpha)\ (\frac{\beta}{1+\beta t})^{\alpha}} d\lambda$$

$$= \frac{1}{(1+\beta t)^{\alpha}} \tag{2}$$

Then the density function of T is given by

$$f(t) = -\frac{dR(t)}{dt}$$

$$= \frac{\alpha\beta}{(1+\beta t)^{\alpha+1}} \qquad (3)$$

Suppose that of N items that were put on test it was observed that $t_1, t_2, \ldots, t_k$ were the life times of k items that had failed before T_o and let $t_{k+1} = t_{k+2}, \ldots = t_N = T_o$ were the life times of items that had not failed. The maximum likelihood estimators of the parameters are derived as follows:

The likelihood function is

$$L(\alpha,\beta) = \prod_{i=1}^{k} f(t_i) \prod_{j=k+1}^{N} R(t_j) \qquad (4)$$

Substituting equations (2) and (3) into (4),

$$L(\alpha,\beta) = \prod_{i=1}^{k} \frac{\alpha\beta}{(1+\beta t_i)^{\alpha+1}} \prod_{j=k+1}^{N} \frac{1}{(1+\beta t_j)^{\alpha}}$$

$$= \frac{(\alpha\beta)^k}{\prod_{i=1}^{k} (1+\beta t_i)^{\alpha+1} \prod_{j=k+1}^{N} (1+\beta t_i)^{\alpha}}$$

$$= \frac{(\alpha\beta)^k}{\prod_{i=1}^{k} (1+\beta t_i) \prod_{i=1}^{N} (1+\beta t_i)^{\alpha}} \qquad (5)$$

Taking the logarithm of $L(\alpha,\beta)$,

$$L_n = \ln L(\alpha,\beta)$$

$$= k \ln(\alpha\beta) - \alpha \sum_{i=1}^{N} \ln(1+\beta t_i) - \sum_{i=1}^{k} \ln(1+\beta t_i) \tag{6}$$

The maximum likelihood estimators of α and β can be obtained by differentiating L_n

$$\frac{\partial L_n}{\partial \alpha} = \frac{k}{\alpha} - \sum_{i=1}^{N} \ln(1+\beta t_i) \tag{7}$$

$$\frac{\partial L_n}{\partial \beta} = \frac{k}{\beta} - \alpha \sum_{i=1}^{N} \frac{t_i}{1+\beta t_i} - \sum_{i=1}^{k} \frac{t_i}{1+\beta t_i} \tag{8}$$

Suppose first that α is known. Then $\hat{\beta}$, the estimate of β can be computed by setting equation (8) to zero

$$\frac{k}{\beta} - \alpha \sum_{i=1}^{N} \frac{t_i}{1+\beta t_i} - \sum_{i=1}^{k} \frac{t_i}{1+\beta t_i} = 0 \tag{9}$$

Rearranging equation (9),

$$\frac{k}{\beta} = \alpha \sum_{i=1}^{N} \frac{t_i}{1+\beta t_i} + \sum_{i=1}^{k} \frac{t_i}{1+\beta t_i}$$

$$k = \alpha \sum_{i=1}^{N} \frac{\beta t_i}{1+\beta t_i} + \sum_{i=1}^{k} \frac{\beta t_i}{1+\beta t_i}$$

$$= \alpha \sum_{i=1}^{N} \left(1-\frac{1}{1+\beta t_i}\right) + \sum_{i=1}^{k} \left(1-\frac{1}{1+\beta t_i}\right)$$

$$= N\alpha - \sum_{i=1}^{N} \frac{1}{1+\beta t_i} + k - \sum_{i=1}^{k} \frac{1}{1+\beta t_i} .$$

Finally,

$$\alpha \sum_{i=1}^{N} \frac{1}{1+\beta t_i} + \sum_{i=1}^{k} \frac{1}{1+\beta t_i} - N\alpha = 0 \qquad (10)$$

The solution of equation (10) is $\hat{\beta}$, an estimate of β.

Note that since

$$A(\beta) = \alpha \sum_{i=1}^{N} \frac{1}{1+\beta t_i} + \sum_{k=1}^{k} \frac{1}{1+\beta t_i} - N\alpha$$

is a decreasing function of β with $A(0) = k$ and

$\lim_{\beta\to\infty} A(\beta) = -N\alpha$, there exists exactly one solution for β whenever $\alpha > 0$ and $N \geq k \geq 1$.

If β is known and β is positive then $\hat{\alpha}$ could be computed directly by setting equation (7) to zero,

$$\hat{\alpha} = \frac{k}{\sum_{i=1}^{N} \ln(1+\beta t_i)} . \qquad (11)$$

If α and β were both unknown, $\hat{\alpha}$ and $\hat{\beta}$ could be obtained by solving the equations simultaneously

$$\frac{k}{\alpha} - \sum_{i=1}^{N} \ln(1+\beta t_i) = 0 \tag{12}$$

$$\frac{k}{\beta} - \alpha \sum_{i=1}^{N} \frac{t_i}{1+\beta t_i} - \sum_{i=1}^{k} \frac{t_i}{1+\beta t_i} = 0 \,. \tag{13}$$

Eliminating α from equations (12) and (13) results in an equation that contains only β, say $B(\beta) = 0$ where

$$B(\beta) = \frac{1}{k} \sum_{i=1}^{N} \ln(1+\beta t_i) \sum_{i=1}^{k} \frac{1}{1+\beta t_i} - \sum_{i=1}^{N} \frac{\beta t_i}{1+\beta t_i} \tag{14}$$

Then $\hat{\beta}$ is the solution of $B(\beta) = 0$ and $\hat{\alpha}$ can be solved using equation (11).

We now proceed to examine the behavior of the function $B(\beta)$. In particular $\hat{\beta}$ exists only under certain conditions. Note that $B(0) = 0$ and $\lim_{\beta\to\infty} B(\beta) = -N$ and $B(\beta)$ is a continuous function. Hence, $B(\beta) = 0$ would have a positive solution if its derivative at $\beta = 0$ is positive. Since $B'(0) = 0$, the sign of $\lim_{\beta\to 0} \frac{B'(\beta)}{\beta}$ may be examined. A condition that $B(\beta) = 0$ has a positive solution is that $\lim_{\beta\to 0} \frac{B'(\beta)}{\beta} > 0$ (see Figure 1).

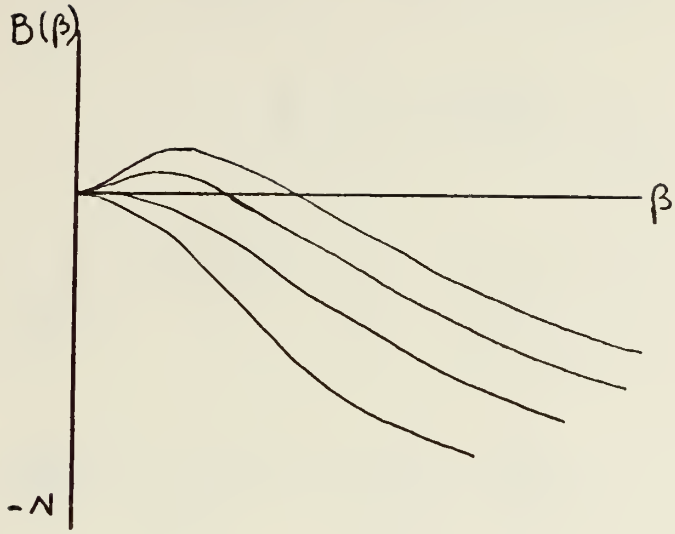

Figure 1

Graph of B(β)

Rewriting equation (14),

$$B(\beta) = \frac{1}{k} \sum_{i=1}^{N} \ln(1+\beta t_i) \sum_{i=1}^{k} \frac{1}{1+\beta t_i} - \sum_{i=1}^{N} \frac{\beta t_i}{1+\beta t_i}$$

$$B'(\beta) = \frac{1}{k} \sum_{i=1}^{N} \frac{t_i}{1+\beta t_i} \sum_{i=1}^{k} \frac{1}{1+\beta t_i} - \frac{1}{k} \ln(1+\beta t_i) \sum_{i=1}^{k} \frac{t_i}{(1+\beta t_i)^2}$$

$$- \sum_{i=1}^{N} \frac{t_i}{1+\beta t_i} + \sum_{i=1}^{N} \frac{\beta t_i^{\,2}}{(1+\beta t_i)^2}$$

$$= \sum_{i=1}^{N} \frac{\beta t_i^{\,2}}{(1+\beta t_i)^2} - \frac{1}{k} \sum_{i=1}^{N} \frac{t_i}{1+\beta t_i} [k - \sum_{i=1}^{k} \frac{1}{1+\beta t_i}]$$

$$- \frac{1}{k} \sum_{i=1}^{N} \ln(1+\beta t_i) \sum_{i=1}^{k} \frac{t_i}{(1+\beta t_i)^2} \qquad (15)$$

$$\frac{B'(\beta)}{\beta} = \sum_{i=1}^{N} \frac{t_i^{\,2}}{(1+\beta t_i)^2} - \frac{1}{k} \sum_{i=1}^{N} \frac{t_i}{1+\beta t_i} \sum_{i=1}^{k} \frac{t_i}{1+\beta t_i}$$

$$- \frac{1}{k} \sum_{i=1}^{N} \ln\frac{(1+\beta t_i)}{\beta} \sum_{i=1}^{k} \frac{t_i}{(1+\beta t_i)^2} \qquad (16)$$

Note that $\lim_{\beta\to 0} \frac{\ln(1+\beta t_i)}{\beta} = t_i$. Taking the limit of equation (16) we get

$$\lim_{\beta\to 0} \frac{B'(\beta)}{\beta} = \sum_{i=1}^{N} t_i^{\,2} - \frac{2}{k} \sum_{i=1}^{N} t_i \sum_{i=1}^{k} t_i \,. \qquad (17)$$

Since it is required that $\lim_{\beta\to 0} \frac{B'(\beta)}{\beta} > 0$ to get a positive solution for $\hat{\beta}$, it can be stated from equation (17) that a sufficient condition for the existence of a positive $\hat{\beta}$ is that

$$\sum_{i=1}^{N} t_i^{\,2} > \frac{2}{k} \sum_{i=1}^{N} t_i \sum_{i=1}^{k} t_i \qquad (18)$$

The reliability estimate $\hat{R}(t)$ could be computed using equation (2) whenever a positive solution for $\hat{\beta}$ exists. If a positive solution for $\hat{\beta}$ does not exist however, $\hat{R}(t)$ can not be computed using this equation. In this latter case, the reliability could be estimated in the following manner.

Assuming Λ is Gamma $G(\alpha,\beta)$ distributed, then $E(\Lambda) = \alpha\beta$. Setting $\alpha\beta = \gamma$ and substituting $\alpha = \gamma/\beta$ into equation (6), the log likelihood function becomes

$$L_n = k \ln \gamma - \frac{\gamma}{\beta} \sum_{i=1}^{N} \ln(1+\beta t_i) - \sum_{i=1}^{k} \ln(1+\beta t_i) \qquad (19)$$

Differentiating L_n we get

$$\frac{\partial L_n}{\partial \gamma} = \frac{k}{\gamma} - \frac{1}{\beta} \sum_{i=1}^{N} \ln(1+\beta t_i) \qquad (20)$$

$$\frac{\partial L_n}{\partial \beta} = -\frac{\gamma}{\beta} \sum_{i=1}^{N} \frac{t_i}{1+\beta t_i} + \frac{\gamma}{\beta^2} \sum_{i=1}^{N} \ln(1+\beta t_i) - \sum_{i=1}^{k} \frac{t_i}{1+\beta t_i} \,. \qquad (21)$$

The M.L.E. of γ and β could be obtained by setting equations (20) and (21) to zero, and solving simultaneously for γ and β. The resulting estimator for γ is

$$\hat{\gamma} = \frac{k\beta}{\sum_{i=1}^{N} \ln(1+\beta t_i)} \qquad (22)$$

Substituting $\hat{\gamma}$ from equation (22) into equation (21) and rearranging,

$$\frac{1}{k} \sum_{i=1}^{k} \frac{1}{1+\beta t_i} \sum_{i=1}^{N} \ln(1+\beta t_i) = \sum_{i=1}^{N} \frac{\beta t_i}{1+\beta t_i}$$

which is exactly the same as equation (14). Thus the M.L.E. of β is the same as before. However whenever a positive solution for $\hat{\beta}$ does not exist the reliability estimate could be computed by treating the life times of the components as exponential random variables with a constant

failure rate $\hat{\gamma}$, that could be computed by taking the limit of equation (22) to get

$$\hat{\gamma} = \lim_{\beta \to 0} \frac{k\beta}{\sum_{i=1}^{N} \ln(1+\beta t_i)} = \frac{k}{\sum_{i=1}^{N} t_i} \tag{23}$$

In this case, the reliability estimate is given by

$$\hat{R}(t) = e^{-\hat{\gamma} t} \tag{24}$$

III. COMPUTER SIMULATION

To determine the robustness of this estimation procedure, computer simulations were conducted for a discrete probability distribution on Λ. Let $\lambda_1, \lambda_2, \ldots, \lambda_m$ be the chosen values of failure rates of the components with associated probabilities $p_1, p_2, \ldots, p_m$. A total of N exponential failure times T's were generated using a standard exponential random number generator. The T's were then truncated at a planned test time T_0. If this set of T's gave any positive solution for $\hat{\beta}$, then $\hat{\beta}$ could be computed by setting equation (14) to zero and solving for β and $\hat{\alpha}$ could be obtained from equation (11). The reliability estimate was computed by using

$$\hat{R}(t) = \frac{k}{(1+\hat{\beta}t)^{\hat{\alpha}}} \tag{25}$$

If no positive $\hat{\beta}$ exists, then the exponential procedure was used. $\hat{\gamma}$ the estimate of $E(\Lambda)$ was computed by using

$$\hat{\gamma} = \frac{k}{\sum_{i=1}^{k} t_i + (N-k)T_0} \tag{26}$$

where

N: the number of components on test

T_0: the planned test time

k: the number of components that failed before T_0.

The reliability estimate then was computed using equation (24).

Since the true reliability of the component is

$$R(t) = \sum_{i=1}^{m} p_i e^{-\lambda_i t} \qquad (27)$$

where m is the number of chosen failure rates, a measure of robustness of this estimate is $\hat{R}(t) - R(t)$.

A delineation of the simulation steps is as follows:

(1) Generate N times to failures $T_1, T_2, \ldots, T_N$ where T_i is exponentially distributed with parameter λ_i and each λ_i is randomly generated in accordance with the stated probability distribution on Λ.

(2) Truncate $T_1, T_2, \ldots, T_N$ at a test time T_0.

(3) Check if this set of $T_1, T_2, \ldots, T_N$ give any positive $\hat{\beta}$ by taking a small value β_0 then check the value of $B(\beta_0)$ from equation (14).

If $B(\beta_0)$ is positive, continue to step (4).

If $B(\beta_0) = 0$, then $\hat{\beta} = \beta_0$, and continue to step (5).

If $B(\beta_0) < 0$, use equation (26) and (24) to compute the reliability estimate then continue to step (7).

(4) Compute $\hat{\beta}$ from $B(\beta) = 0$ using Newton-Raphson method.

(5) Compute $\hat{\alpha}$ using equation (11).

(6) Compute the reliability estimate $\hat{R}(t)$ using equation (25).

(7) Replicate steps (1) through (6) r times.

(8) Compute the mean value and standard deviation of all values of $\hat{R}(t)$, and let $\hat{R}(t)$ be the mean value of $\hat{R}(t)$.

(9) Compute the true reliability R(t) by using equation (27).

(10) The measure of robustness is $\hat{R}(t) - R(t)$.

A second simulation was done as a comparison to the first one. The procedure was similar to the first one except that instead of truncating the failure times $T_1, T_2, \ldots, T_N$ at a test time T_0 (see step 2), they were truncated at the time of the k^{th} failure. In other words N components were put on test, and the test was terminated after k components had failed.

Using this procedure however, a positive solution for $\hat{\beta}$ will never exist for $k \leq 2$. Only if $k \geq 3$ under certain conditions will a positive $\hat{\beta}$ exist. This can be shown as follows.

Recall that a sufficient condition for the existence of a positive $\hat{\beta}$ (see inequality (18)) is,

$$\sum_{i=1}^{N} t_i^2 > \frac{2}{k} \sum_{i=1}^{N} t_i \sum_{i=1}^{k} t_i$$

For k=1 then $t_N=t_{N-1}=\ldots=t_2=t_1$ and the inequality (18) becomes

$$Nt_1^2 > 2Nt_1^2$$

and this, of course is invalid.

For k=2, $t_N=t_{N-1}=\ldots=t_2$ and the inequality (18) becomes

$$t_1^2 + (N-1)t_2^2 > t_1^2 + (N-1)t_2^2 + Nt_1t_2$$

and this is also invalid.

For k=3, $t_N=t_{N-1}=\ldots=t_3$, the inequality (18) becomes

$$t_1^2 + t_2^2 + (N-2)t_3^2 > \frac{2}{3}(t_1+t_2+(N-2)t_3)(t_1+t_2+t_3)$$

or

$$\frac{1}{3}[t_1^2 + t_2^2 + (N-2)t_3^2] > \frac{2}{3}[2t_1t_2+(N-1)t_1t_3+(N-1)t_2t_3]$$

and this inequality could be valid for some sets of t_i's.

Thus, using this later simulation procedure $\hat{\beta} > 0$ will possibly exist only for $k \geq 3$.

Using these two procedures, simulations were done and the results appear in Section IV.

IV. RESULTS OF THE SIMULATION

Graphs which depict the results of the simulation appear on the following pages. Symbols used on the graphs are:

$R(t)$:	The reliability at time t
$\hat{R}(t)$:	The estimate of $R(t)$
N:	Number of components on test
T_o:	The planned test time
r:	Number of replications
λ:	Parameters of generated life time T's of the components
p:	Discrete probability distribution for λ
k:	Number of components that failed before T_o the planned test time
PB:	Number of replications that gave positive solutions for $\hat{\beta}$
NB:	Number of replications that did not give a positive solution for $\hat{\beta}$
Total:	Total number of replications that had k failures

To illustrate the graphs, let us observe Figure 2. This graph shows that five exponential life times (N=5) were generated via a set of λ: .010, .010 with probability density p: .50, .50. By using a planned test time T_o = 10, the reliability estimate was computed. With 50 replications (r=50) the average reliability $\hat{R}(t)$ was computed and was shown on the graph. Further, it can be observed that for k = 0 (no failure occurred), NB = 28 (28 replications

occurred without any failure). It happens that 3 replications occurred with 2 failures (Total = 3, k = 2) and all of these three replications gave no positive $\hat{\beta}$ (NB=3) and of course no replication gave positive solutions for $\hat{\beta}$ (PB=0).

Another example is Figure 25 where 50 replications (r=50) of 10 exponential life times (N=10) were generated via a set of λ: .005, .020 with probability density p: .50, .50. It is observed that by using a planned test time T_o = 20 unit times, 22 replications (Total=22) occurred with 2 failures (k=2) of which 5 replications gave positive solutions for $\hat{\beta}$ (PB=5) and the rest did not give any positive solution for $\hat{\beta}$ (NB=17).

Figure 44 shows that 50 replications (r=50) of 10 exponential life times (N=10) were generated via a set of λ: .005, .010, .015, .020 with probability density p: .40, .20, .35, .05. The N items were tested until a fixed number of failures (k=5) occurred. Eight of those replications gave positive solutions for $\hat{\beta}$ (PB=8) and the rest (NB=42) gave no positive solution for $\hat{\beta}$.

TABLE OF GRAPHS

T_o	N	λ		p		Fig. #
10	5	.01	.01	.50	.50	2
10	50	.01	.01	.50	.50	3
10	5	.01	.02	.50	.50	4
10	50	.01	.02	.50	.50	5
10	5	.005	.02	.50	.50	6
10	50	.005	.02	.50	.50	7
10	5	.01	.05	.05	.40	8
		.02	.06	.10	.25	
		.04		.20		
10	50	.01	.05	.05	.40	9
		.02	.06	.10	.25	
		.04		.20		
10	5	.005	.015	.20	.40	10
		.010	.020	.20	.20	
10	50	.005	.015	.20	.40	11
		.010	.020	.20	.20	
10	5	.01	.10	.05	.40	12
		.05	.15	.10	.25	
		.08		.20		
10	50	.01	.10	.05	.40	13
		.05	.15	.10	.25	
		.08		.20		
10	5	.005	.020	.10	.10	14
		.007	.025	.10	.10	
		.010	.027	.10	.10	
		.015	.030	.10	.10	
		.017	.035	.10	.10	

TABLE OF GRAPHS
(cont.)

T_o	N	λ		p		Fig. #
10	50	.005	.020	.10	.10	15
		.007	.025	.10	.10	
		.010	.027	.10	.10	
		.015	.030	.10	.10	
		.017	.035	.10	.10	
10	5	.005	.020	.03	.20	16
		.007	.025	.05	.07	
		.010	.027	.10	.05	
		.015	.030	.20	.03	
		.017	.035	.25	.02	
10	50	.005	.020	.03	.20	17
		.007	.025	.05	.07	
		.010	.027	.10	.05	
		.015	.030	.20	.03	
		.017	.035	.25	.02	
10	5	.005	.020	.25	.05	18
		.007	.025	.20	.05	
		.010	.027	.20	.03	
		.015	.030	.10	.03	
		.017	.035	.07	.02	
10	50	.005	.020	.25	.05	19
		.007	.025	.20	.05	
		.010	.027	.20	.03	
		.015	.030	.10	.03	
		.017	.035	.07	.02	
20	5	.01	.01	.50	.50	20
20	50	.01	.01	.50	.50	21
20	5	.01	.02	.50	.50	22

TABLE OF GRAPHS
(cont.)

T_o	N	λ		p		Fig. #
20	50	.01	.02	.50	.50	23
20	5	.005	.02	.50	.50	24
20	10	.005	.02	.50	.50	25
20	50	.005	.02	.50	.50	26
20	5	.01	.05	.05	.40	27
		.02	.06	.10	.25	
		.04		.20		
20	50	.01	.05	.05	.40	28
		.02	.06	.10	.25	
		.04		.20		
20	5	.005	.015	.20	.40	29
		.010	.020	.20	.20	
20	50	.005	.015	.20	.40	30
		.010	.020	.20	.20	
20	5	.01	.10	.05	.40	31
		.05	.15	.10	.25	
		.08		.20		
20	50	.01	.10	.05	.40	32
		.05	.15	.10	.25	
		.08		.20		
20	5	.005	.020	.10	.10	33
		.007	.025	.10	.10	
		.010	.027	.10	.10	
		.015	.030	.10	.10	
		.017	.035	.10	.10	
20	50	.005	.020	.10	.10	34
		.007	.025	.10	.10	
		.010	.027	.10	.10	
		.015	.030	.10	.10	
		.017	.035	.10	.10	

TABLE OF GRAPHS
(cont.)

T_o	N	λ		p		Fig. #
20	5	.005	.020	.03	.20	35
		.007	.025	.05	.07	
		.010	.027	.10	.05	
		.015	.030	.20	.03	
		.017	.035	.25	.02	
20	50	.005	.020	.03	.20	36
		.007	.025	.05	.07	
		.010	.027	.10	.05	
		.015	.030	.20	.03	
		.017	.035	.25	.02	
20	5	.005	.020	.25	.05	37
		.007	.025	.20	.05	
		.010	.027	.20	.03	
		.015	.030	.10	.03	
		.017	.035	.07	.02	
20	10	.005	.020	.25	.05	38
		.007	.025	.20	.05	
		.010	.027	.20	.03	
		.015	.030	.10	.03	
		.017	.035	.07	.02	
20	20	.005	.020	.25	.05	39
		.007	.025	.20	.05	
		.070	.027	.20	.03	
		.015	.030	.10	.03	
		.017	.035	.07	.02	

TABLE OF GRAPHS
(cont.)

T_o	N	λ		p		Fig. #
20	50	.005	.020	.25	.05	40
		.007	.025	.20	.05	
		.070	.027	.20	.03	
		.015	.030	.10	.03	
		.017	.035	.07	.02	

k	N	λ		p		Fig. #
3	10	.005	.020	.50	.50	41
5	10	.005	.020	.50	.50	42
3	10	.005	.015	.40	.35	43
		.010	.020	.20	.05	
5	10	.005	.015	.40	.35	44
		.010	.020	.20	.05	
3	10	.005	.020	.25	.05	45
		.007	.025	.20	.05	
		.010	.027	.20	.03	
		.015	.030	.10	.03	
		.017	.035	.07	.02	
5	10	.005	.020	.25	.05	46
		.007	.025	.20	.05	
		.010	.027	.20	.03	
		.015	.030	.10	.03	
		.017	.035	.07	.02	

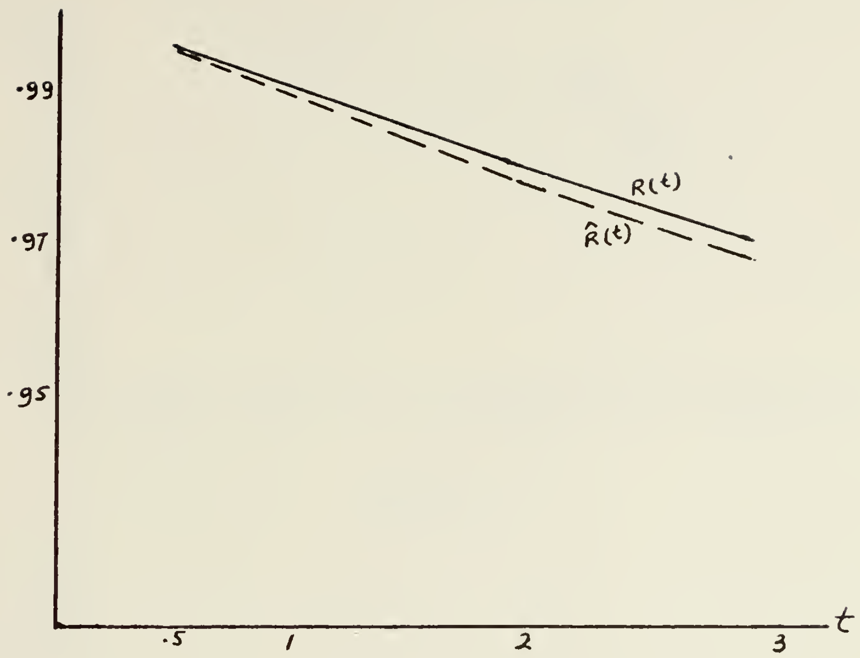

Figure 2

N = 5

T_o = 10

r = 50

λ:	.010	.010
p:	.5	.5

k:	0	1	2	≥3
PB:	0	0	0	0
NB:	28	19	3	0
Total:	28	19	3	0

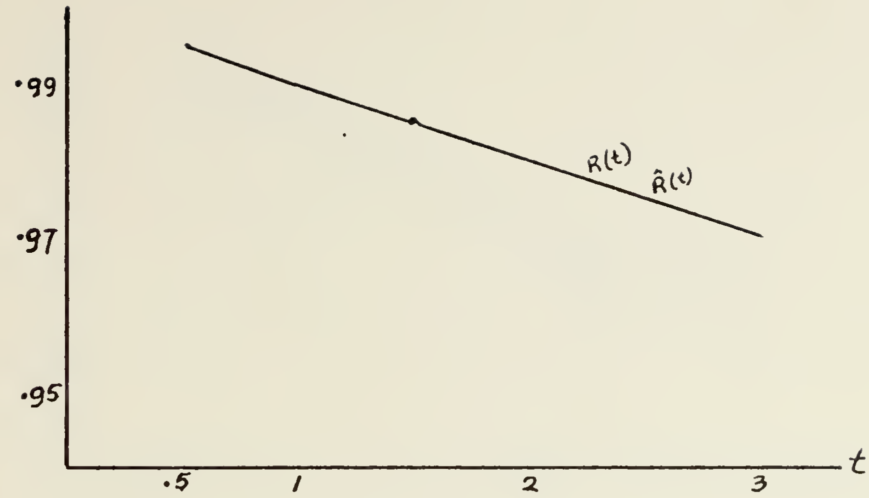

Figure 3

N = 50

T_o = 10

r = 50

λ:	.01	.01
p:	.5	.5

k:	2	3	4	5	6	7	8	9	10
PB:	0	0	0	0	0	0	0	0	0
NB:	6	7	9	11	8	4	1	4	0
Total:	6	7	9	11	8	4	1	4	0

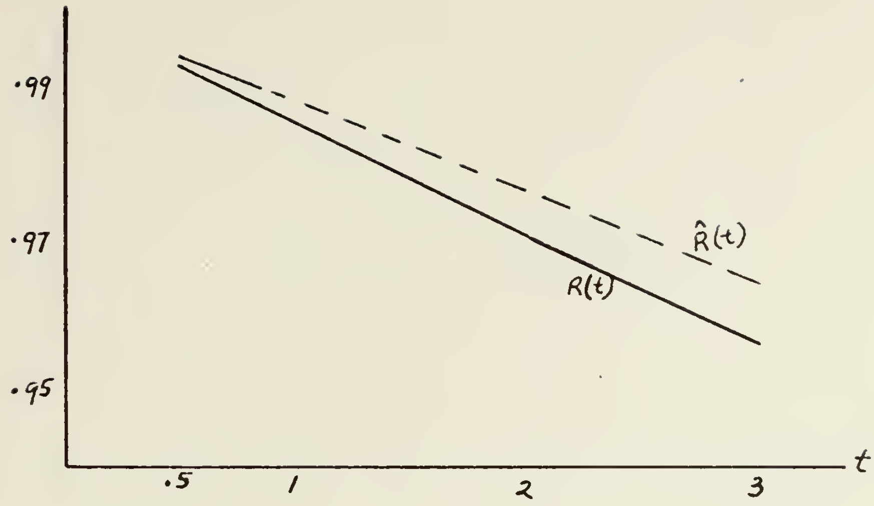

Figure 4

$N = 5$

$T_o = 10$

$r = 50$

λ:	.010	.020
p:	.5	.5

k:	0	1	2	≥3
PB:	0	0	0	0
NB:	28	17	5	0
Total:	28	17	5	0

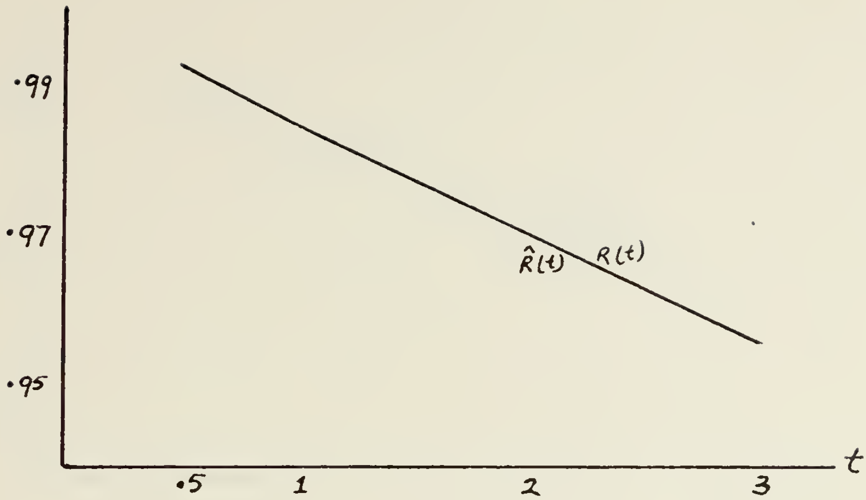

Figure 5

$N = 50$

$T_o = 10$

$r = 50$

λ:	.01	.02
p:	.5	.5

k:	3	4	5	6	7	8	9	10	11	12
PB:	0	0	0	0	0	0	0	0	0	0
NB:	4	1	7	13	6	6	4	4	2	3
Total:	4	1	7	13	6	6	4	4	2	3

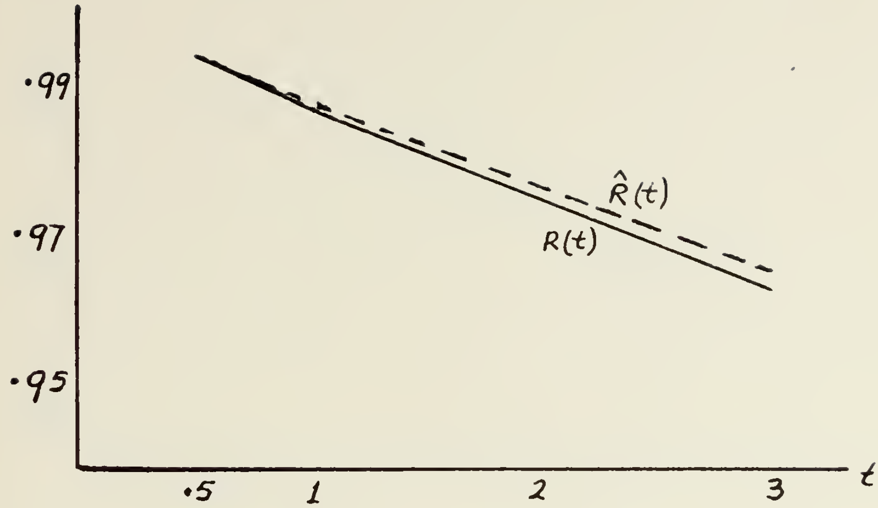

Figure 6

N = 5

T_o = 10

r = 50

λ:	.005	.020
p:	.5	.5

k:	0	1	2	≥3
PB:	0	0	0	0
NB:	29	16	5	0
Total:	29	16	5	0

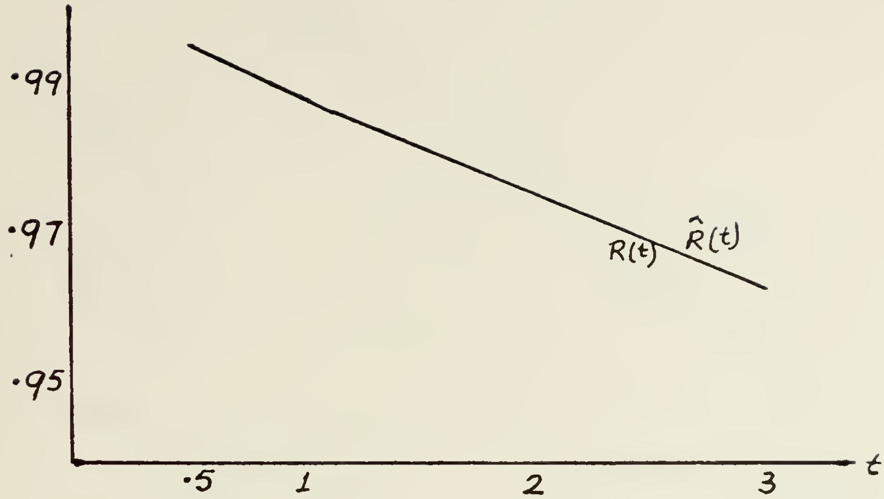

Figure 7

N = 50

T_o = 10

r = 50

λ:	.005	.020
p:	.5	.5

k:	0	1	2	3	4	5	6	7	8	9	10	11	12
PB:	0	0	0	0	0	0	0	0	0	0	0	0	0
NB:	0	0	4	5	9	7	6	8	7	1	0	2	1
TOTAL:	0	0	4	5	9	7	6	8	7	1	0	2	1

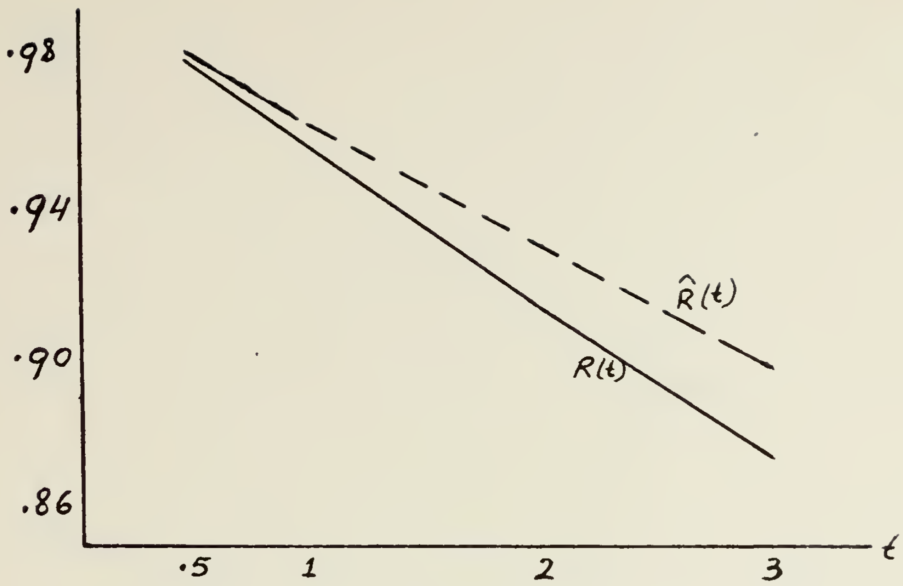

Figure 8

N = 5

T_o = 10

r = 50

λ:	.01	.02	.04	.05	.06
p:	.05	.10	.20	.40	.25

k:	0	1	2	3	4
PB:	0	0	0	0	0
NB:	9	21	13	5	2
Total:	9	21	13	5	2

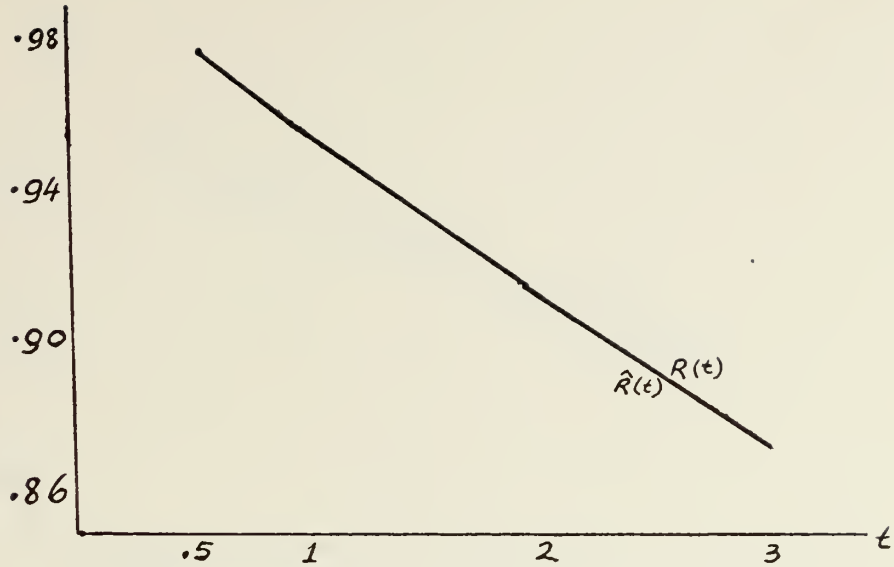

Figure 9

N = 50

T_o = 10

r = 50

λ:	.01	.02	.04	.05	.06
p:	.05	.10	.20	.40	.25

k:	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
PB:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
NB:	2	2	1	7	5	6	7	4	4	2	3	5	0	0	2
Total:	2	2	1	7	5	6	7	4	4	2	3	5	0	0	2

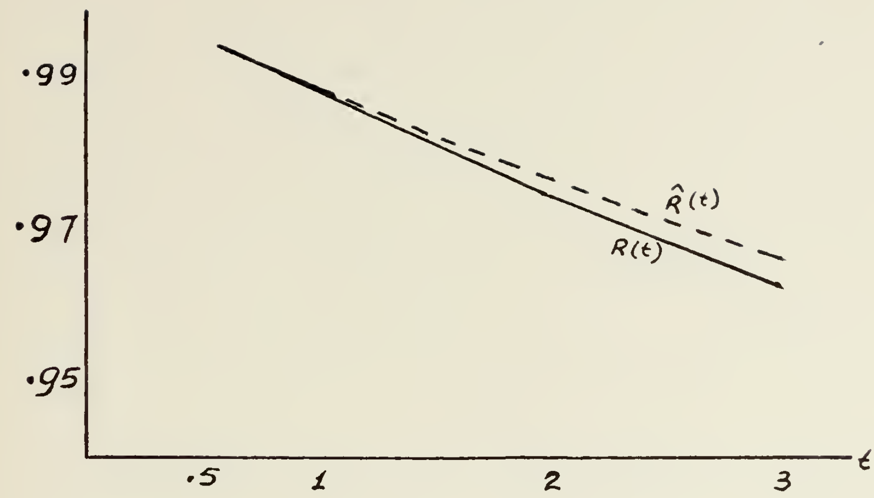

Figure 10

N = 5
T_o = 10
r = 50

λ:	.005	.015	.010	.015	.020
p:	.2	.2	.2	.2	.2

k:	0	1	2	≥ 3
PB:	0	0	0	0
NB:	29	16	5	0
Total:	29	16	5	0

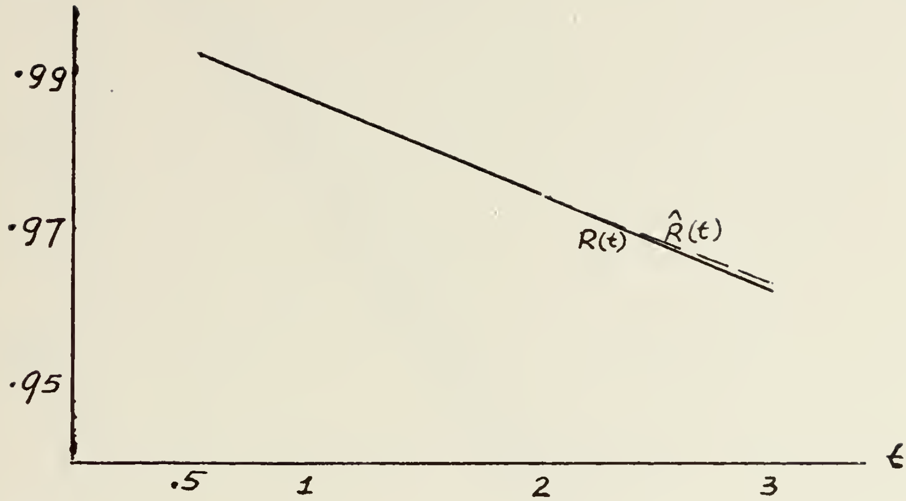

Figure 11

N = 50

T_o = 10

r = 50

λ:	.005	.015	.010	.015	.020
p:	.2	.2	.2	.2	.2

k:	2	3	4	5	6	7	8	9	10	11	12
PB:	0	0	0	0	0	0	0	0	0	0	0
NB:	2	4	6	11	10	9	3	1	3	0	1
Total:	2	4	6	11	10	9	3	1	3	0	1

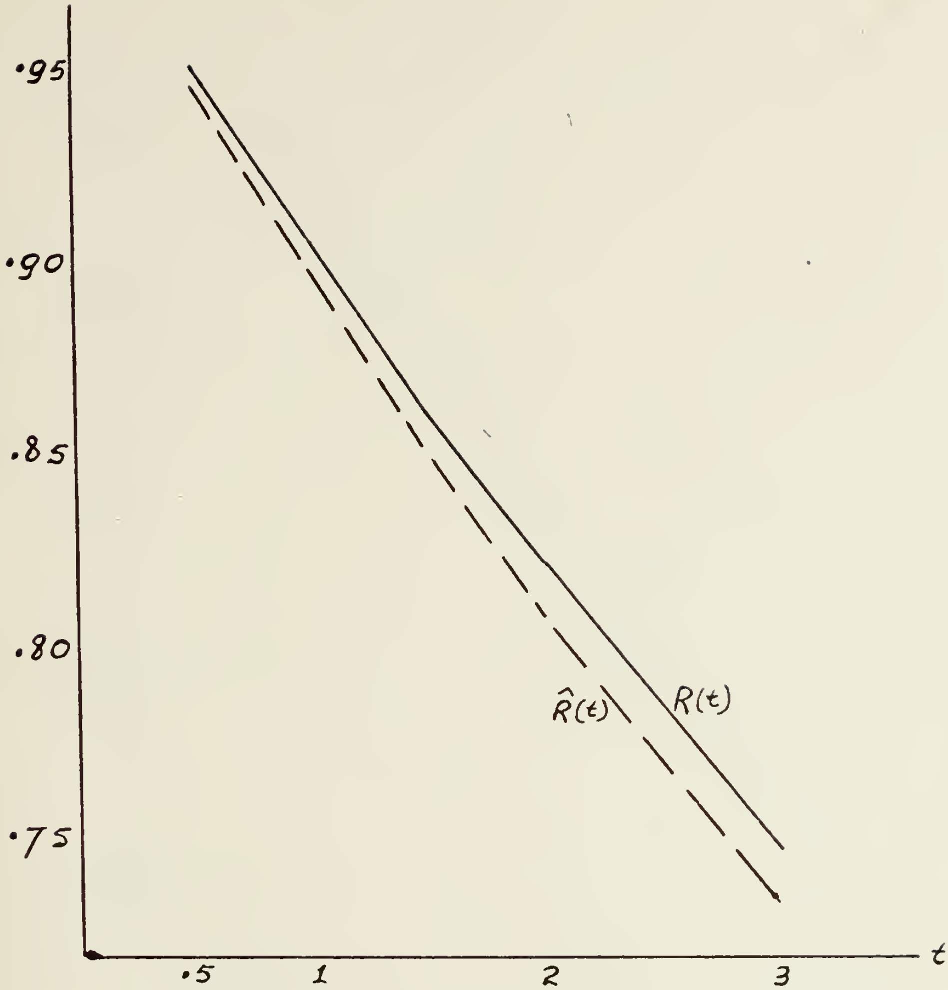

Figure 12

$N = 5$

$T_o = 10$

$r = 50$

λ:	.01	.05	.08	.10	.15
p:	.05	.10	.20	.40	.25

k:	0	1	2	3	4	5
PB:	0	0	0	0	0	0
NB:	3	3	10	13	18	3
Total:	3	3	10	13	18	3

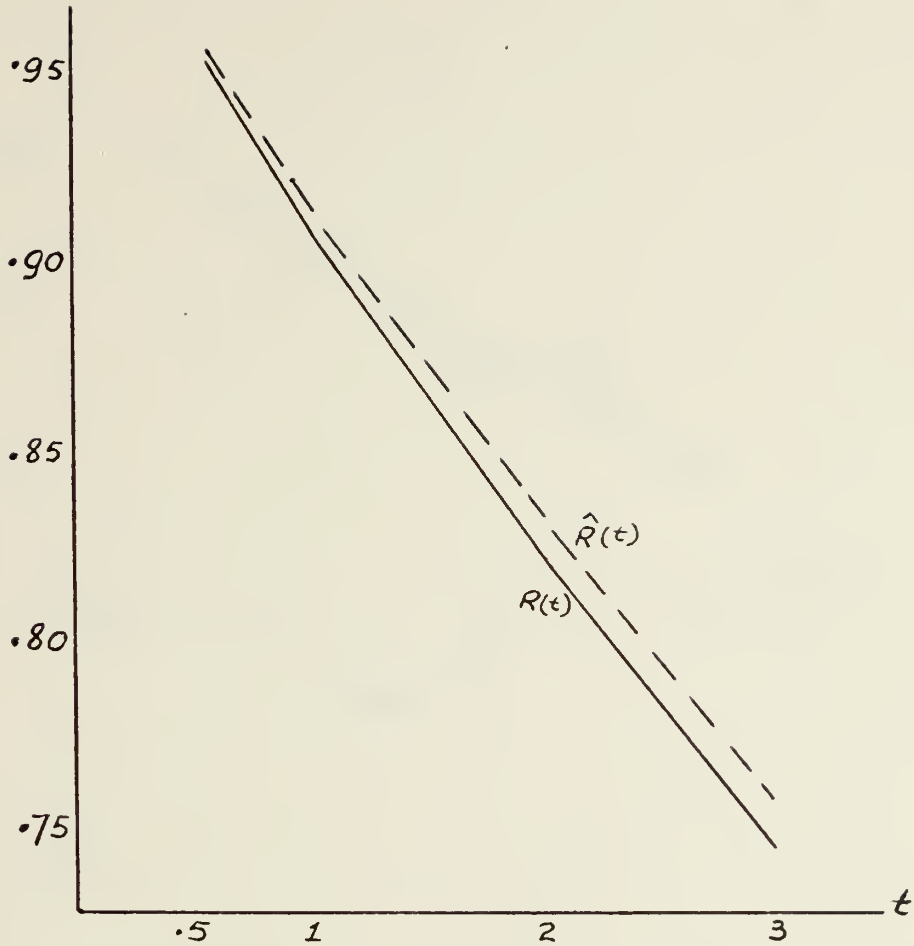

Figure 13

N = 50
T_o = 10
r = 50

λ:	.01	.05	.08	.10	.15
p:	.05	.10	.20	.40	.25

k:	22	23	24	25	26	27	28	29	30	31	32	33	34
PB:	0	0	0	0	0	0	0	0	0	0	0	0	0
NB:	1	2	0	0	4	3	6	8	7	7	7	2	3
Total:	1	2	0	0	4	3	6	8	7	7	7	2	3

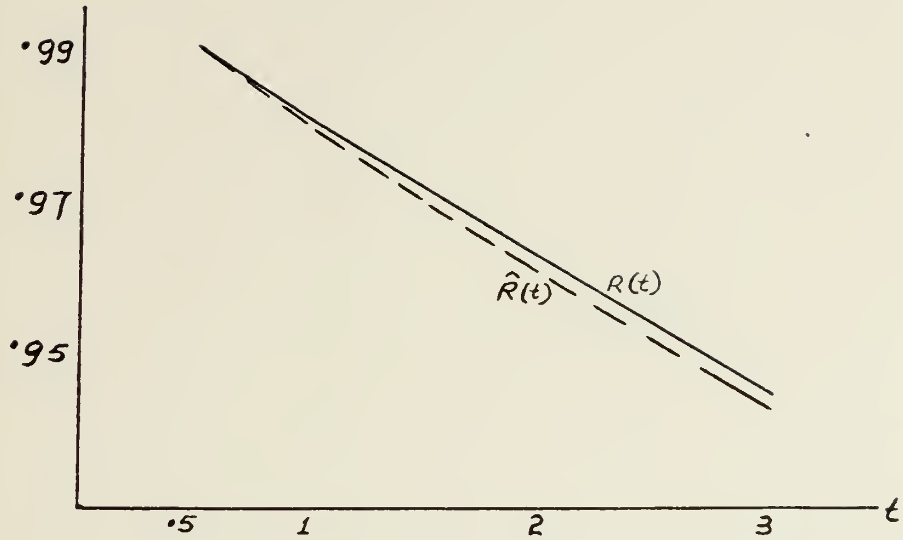

Figure 14

N = 5

T_o = 10

r = 50

λ:	.005	.007	.010	.015	.017	.020	.025	.027	.030	.035
p:	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1

k:	0	1	2
PB:	0	0	0
NB:	19	19	12
Total:	19	19	12

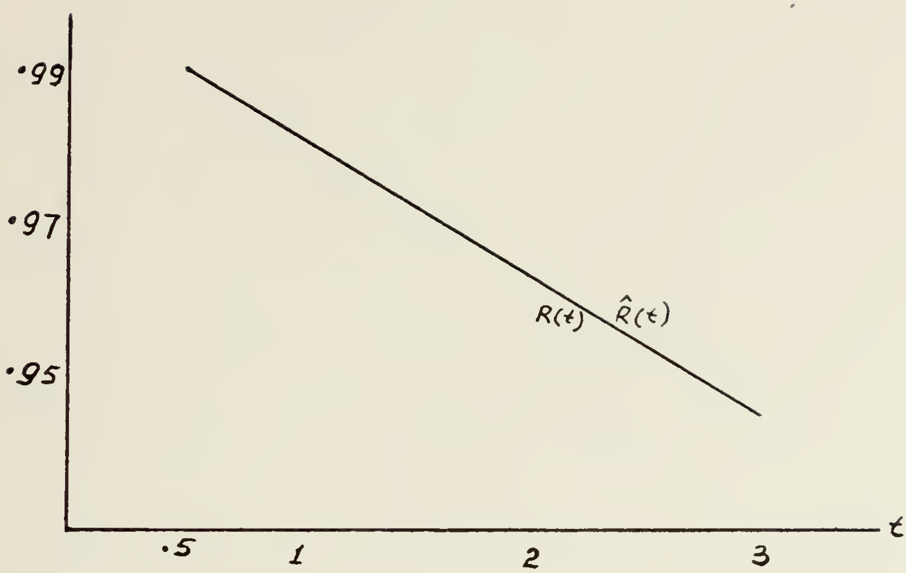

Figure 15

N = 50

T_o = 10

r = 50

λ:	.005	.007	.010	.015	.017	.020	.025	.027	.030	.035
p:	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1

k:	2	3	4	5	6	7	8	9	10	11	12	13	–	17
PB:	0	0	0	0	0	0	0	0	0	0	0	0	0	0
NB:	1	1	1	1	7	7	9	5	9	4	1	3	0	1
Total:	1	1	1	1	7	7	9	5	9	4	1	3	0	1

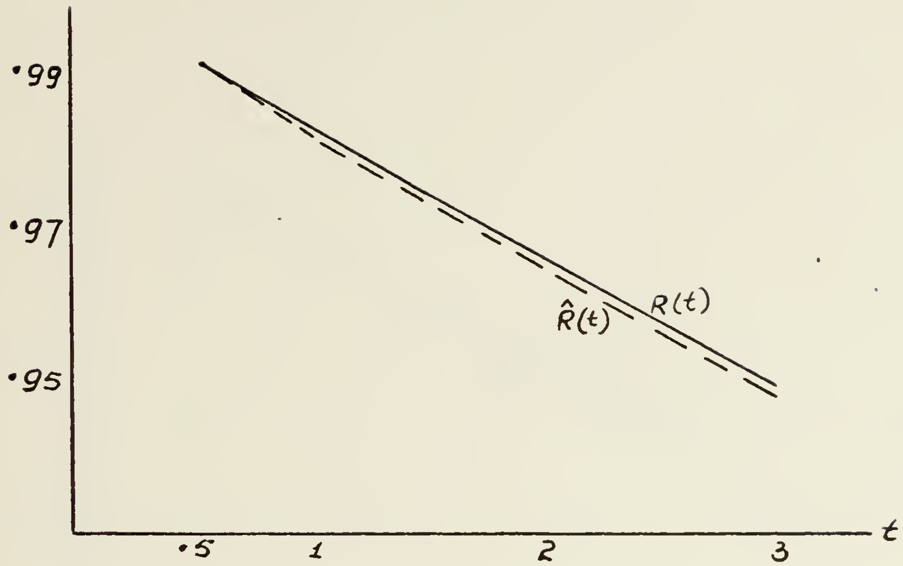

Figure 16

N = 5

T_o = 10

r = 50

λ:	.005	.007	.010	.015	.017	.020	.025	.027	.030	.035
p:	.03	.05	.10	.20	.25	.20	.07	.05	.03	.02

k:	0	1	2	3
PB:	0	0	0	0
NB:	22	14	9	1
Total:	22	14	9	1

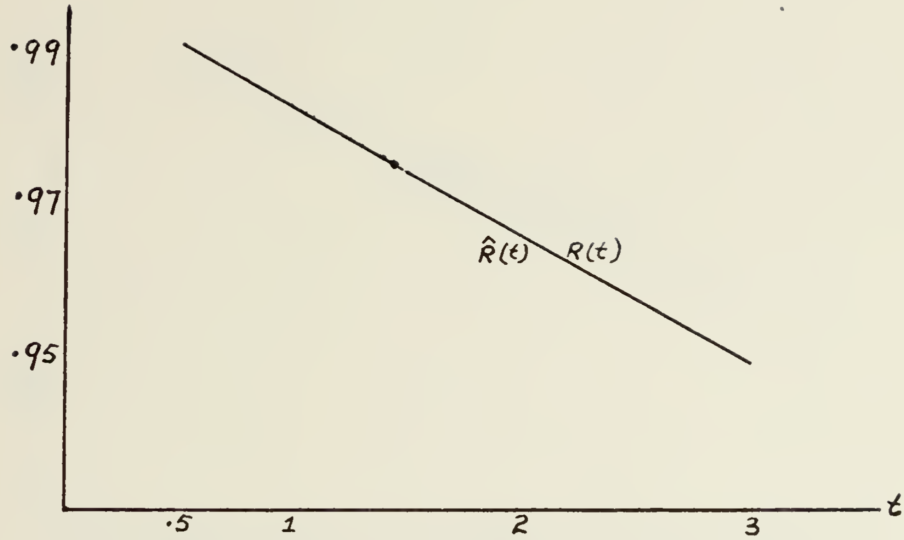

Figure 17

$N = 50$

$T_o = 10$

$r = 50$

λ:	.005	.007	.010	.015	.017	.020	.025	.027	.030	.035
p:	.03	.05	.10	.20	.25	.20	.07	.05	.03	.02

k:	2	3	4	5	6	7	8	9	10	11	12	13	14
PB:	0	0	0	0	0	0	0	0	0	0	0	0	0
NB:	1	1	2	6	6	7	8	5	3	4	4	2	1
Total:	1	1	2	6	6	7	8	5	3	4	4	2	1

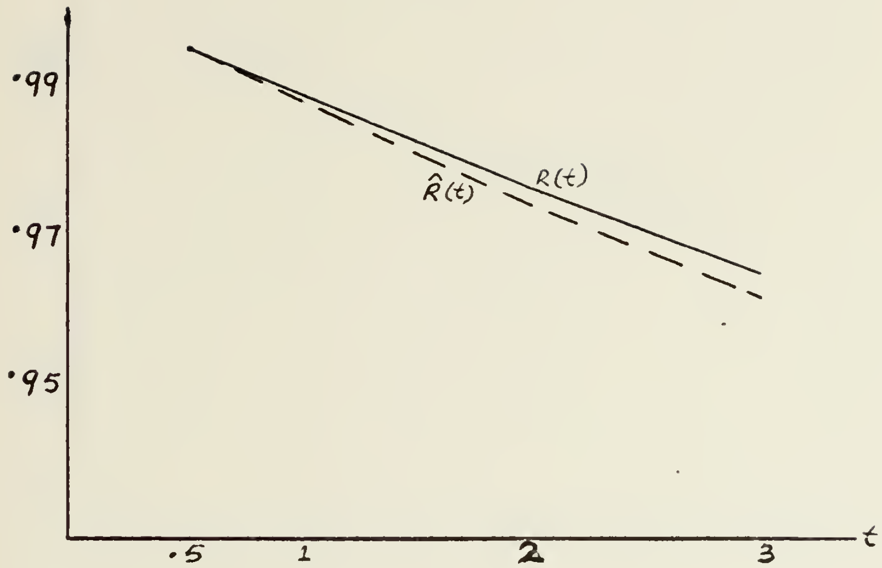

Figure 18

$N = 5$

$T_o = 10$

$r = 50$

λ:	.005	.007	.010	.015	.017	.020	.025	.027	.030	.035
p:	.25	.20	.20	.10	.07	.05	.05	.03	.03	.02

k:	0	1	2
PB:	0	0	0
NB:	25	21	4
Total:	25	21.	4

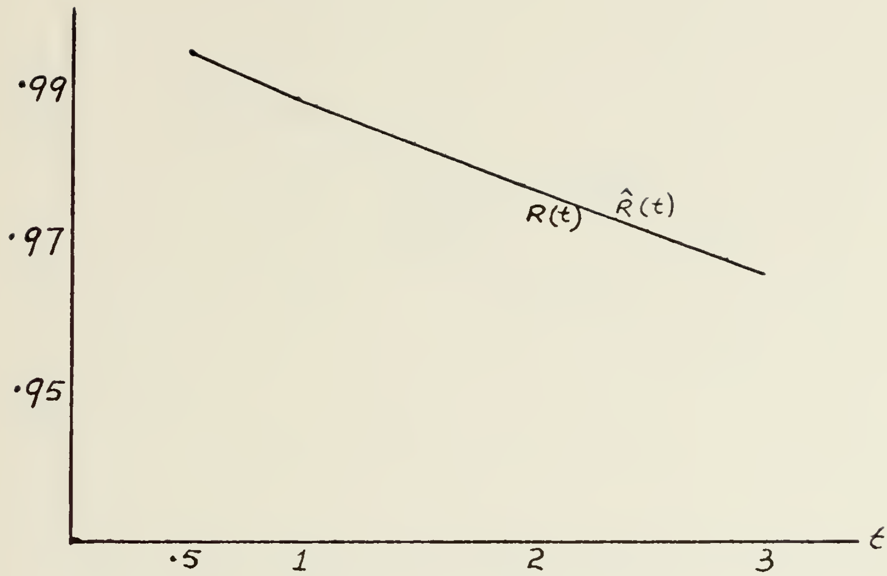

Figure 19

N = 50

T_o = 10

r = 50

λ:	.005	.007	.010	.015	.017	.020	.025	.027	.030	.035
p:	.25	.20	.20	.10	.07	.05	.05	.03	.03	.02

k:	2	3	4	5	6	7	8	9	10
PB:	0	0	0	0	0	0	0	0	0
NB:	5	5	8	8	6	8	4	4	2
Total:	5	5	8	8	6	8	4	4	2

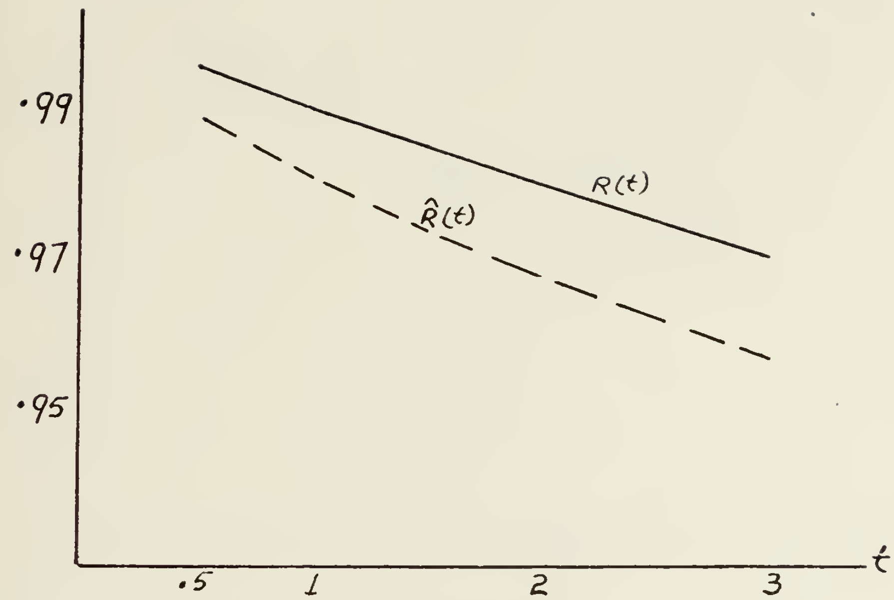

Figure 20

$N = 5$

$T_o = 20$

$r = 50$

λ:	.01		.01		
p:	.5		.5		

k:	0	1	2	3	≥4
PB:	0	12	2	1	0
NB:	17	15	2	1	0
Total:	17	27	4	2	0

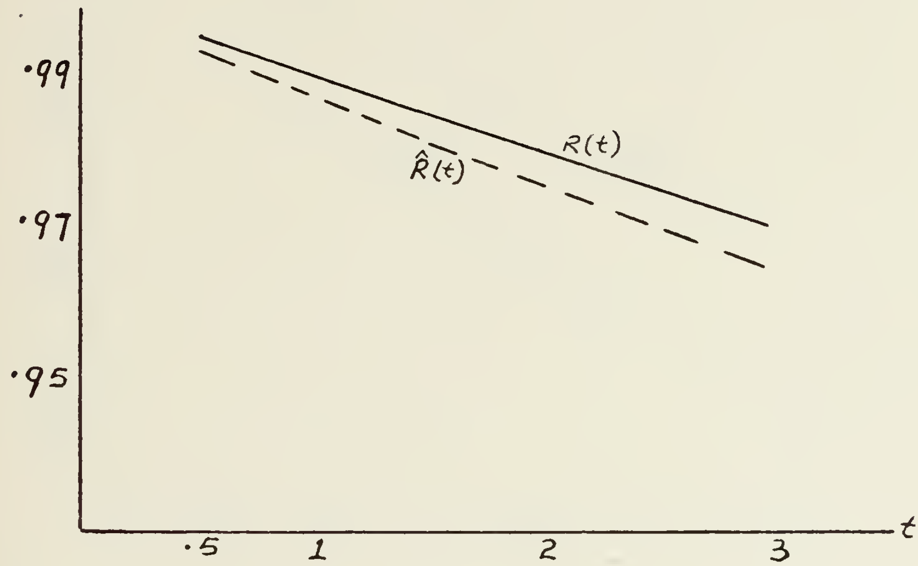

Figure 21

$N = 50$

$T_o = 20$

$r = 50$

λ:	.01	.01
p:	.5	.5

k:	4	5	6	7	8	9	10	11	12	13	14	15	16
PB:	2	1	2	1	2	2	0	1	1	1	0	0	0
NB:	1	0	1	3	6	6	8	5	3	2	0	1	1
Total:	3	1	3	4	8	8	8	6	4	3	0	1	1

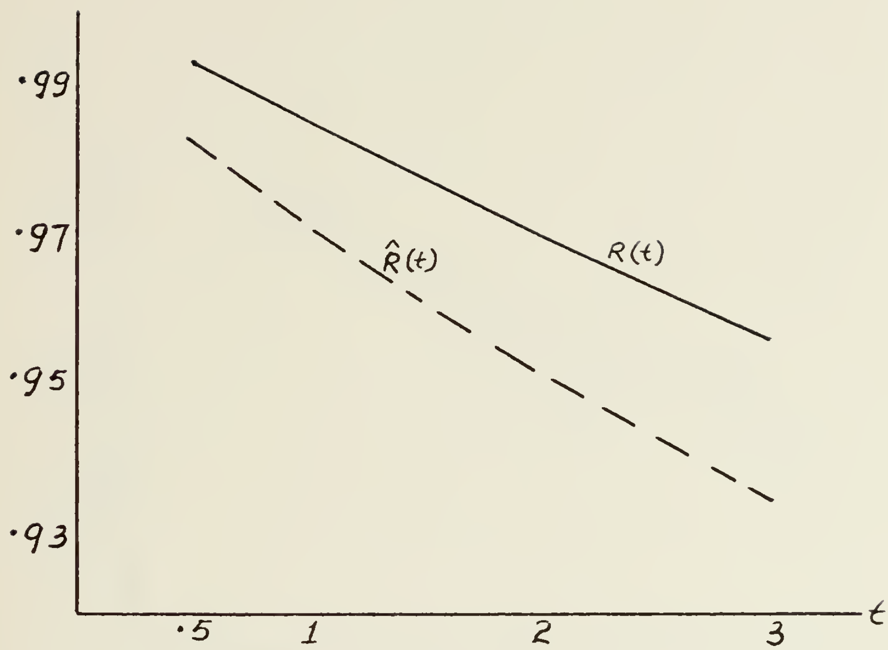

Figure 22

N = 5

T_o = 20

r = 50

λ:	.010	.020
p:	.5	.5

k:	0	1	2	3	4	≥5
PB:	0	5	5	0	0	0
NB:	17	6	10	5	2	0
Total:	17	11	15	5	2	0

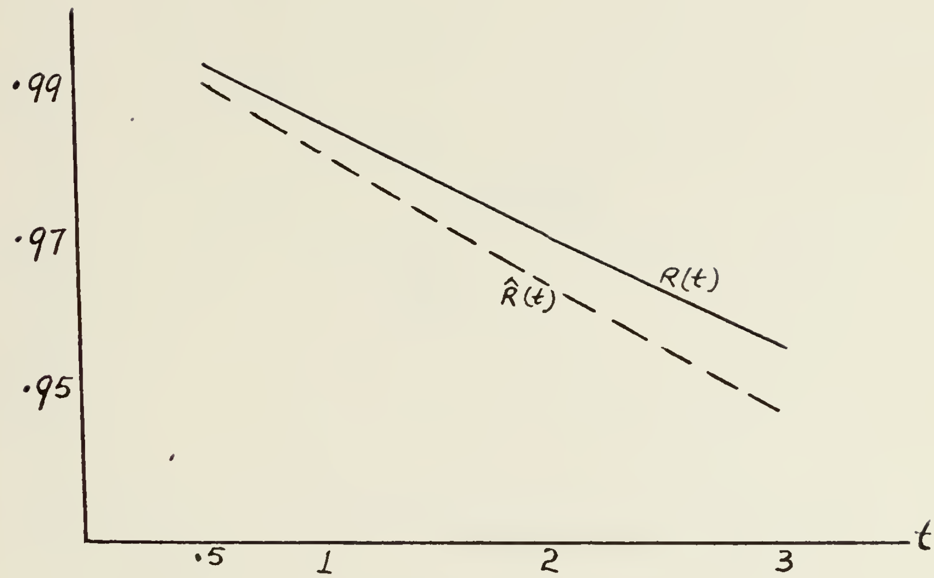

Figure 23

N = 50

T_o = 20

r = 50

λ:	.01	.02
p:	.5	.5

k:	7	8	9	10	11	12	13	14	15	16	17	18	19
PB:	1	0	0	1	3	2	1	1	0	2	0	0	0
NB:	0	3	1	2	5	8	2	6	0	6	3	1	2
Total:	1	3	1	3	8	10	3	7	0	8	3	1	2

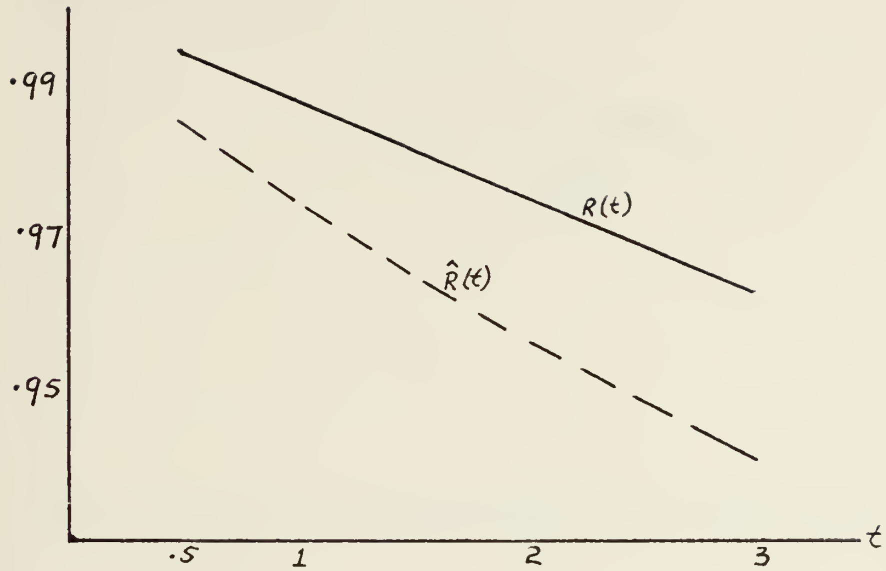

Figure 24

$N = 5$
$T_o = 20$
$r = 50$

λ:	.005	.020
p:	.5	.5

k:	0	1	2	3	≥ 4
PB:	0	5	4	1	0
NB:	16	12	9	3	0
Total:	16	17	13	4	0

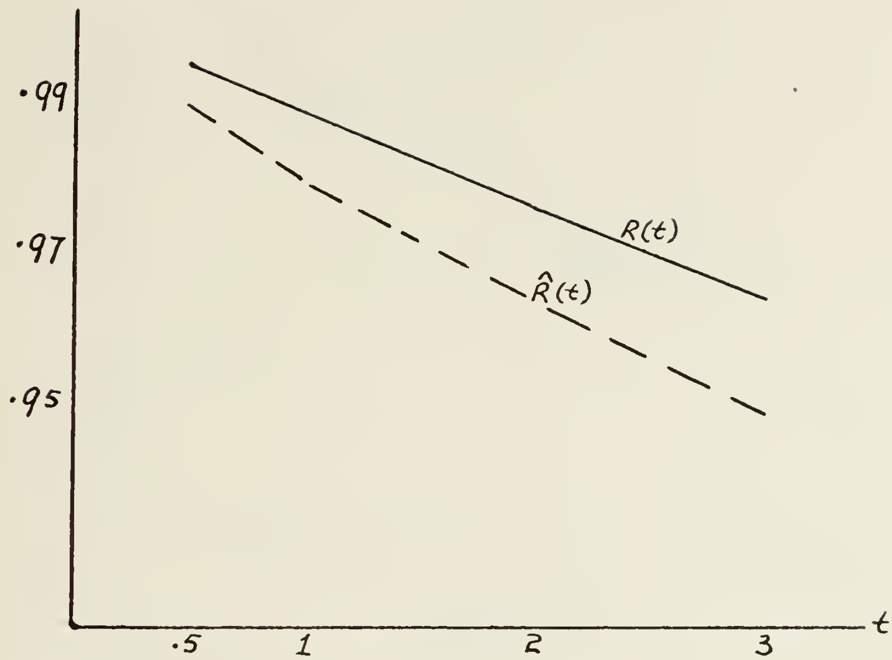

Figure 25

$N = 10$
$T_o = 20$
$r = 50$

λ:	.005	.020
p:	.50	.50

k:	0	1	2	3	4	5
PB:	0	4	5	1	0	1
NB:	10	4	17	3	5	0
Total:	10	8	22	4	5	1

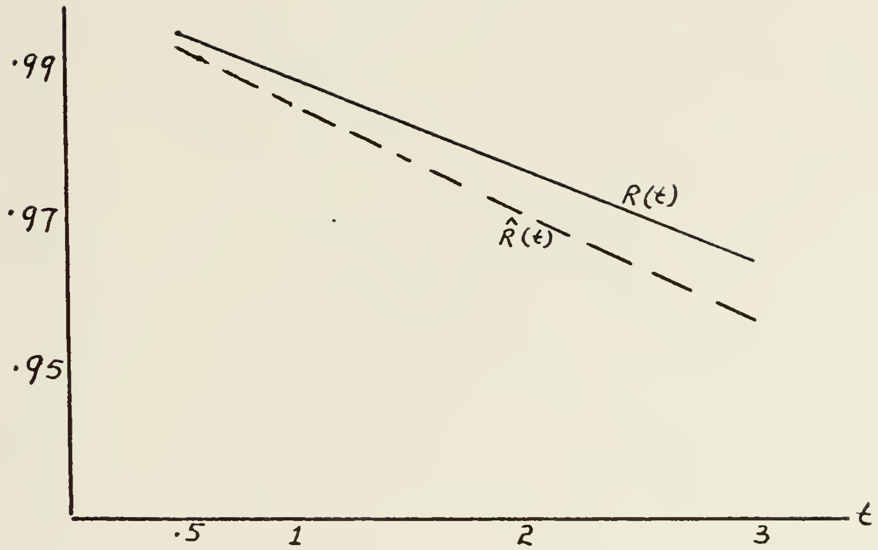

Figure 26

N = 50

T_o = 20

r = 50

λ:	.005	.020
p:	.5	.5

k:	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
PB:	0	0	0	0	4	0	0	1	2	2	2	0	0	0	0
NB:	1	0	0	0	1	2	7	2	8	5	4	3	4	1	1
Total:	1	0	0	0	5	2	7	3	10	7	6	3	4	1	1

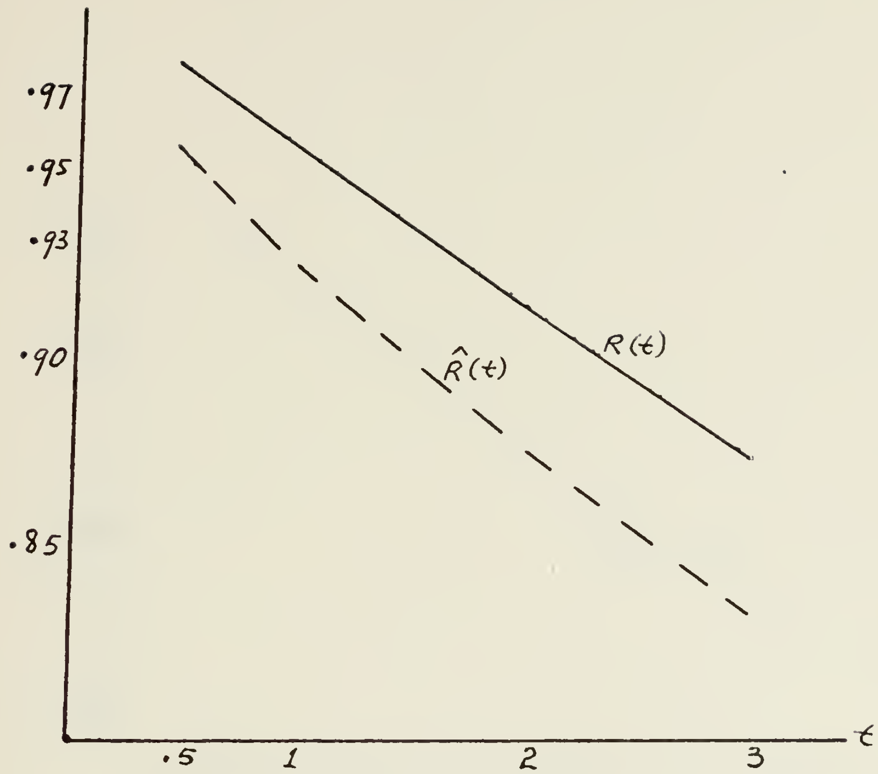

Figure 27

N = 5

T_o = 20

r = 50

λ:	.01	.02	.04	.05	.06
p:	.05	.10	.20	.40	.25

k:	0	1	2	3	4	5
PB:	0	2	6	3	0	0
NB:	0	5	7	8	16	3
Total:	0	7	13	11	16	3

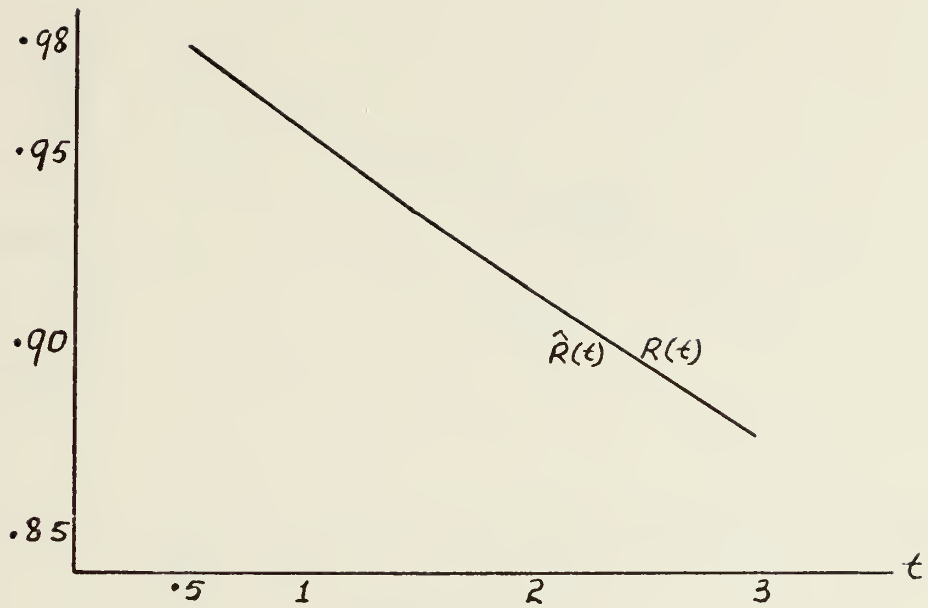

Figure 28

$N = 50$

$T_o = 20$

$r = 50$

λ:	.01	.02	.04	.05	.06
p:	.05	.10	.20	.40	.25

k:	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
PB:	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
NB:	1	1	0	2	6	4	7	4	7	5	4	2	2	3	1
Total:	2	1	0	2	6	4	7	4	7	5	4	2	2	3	1

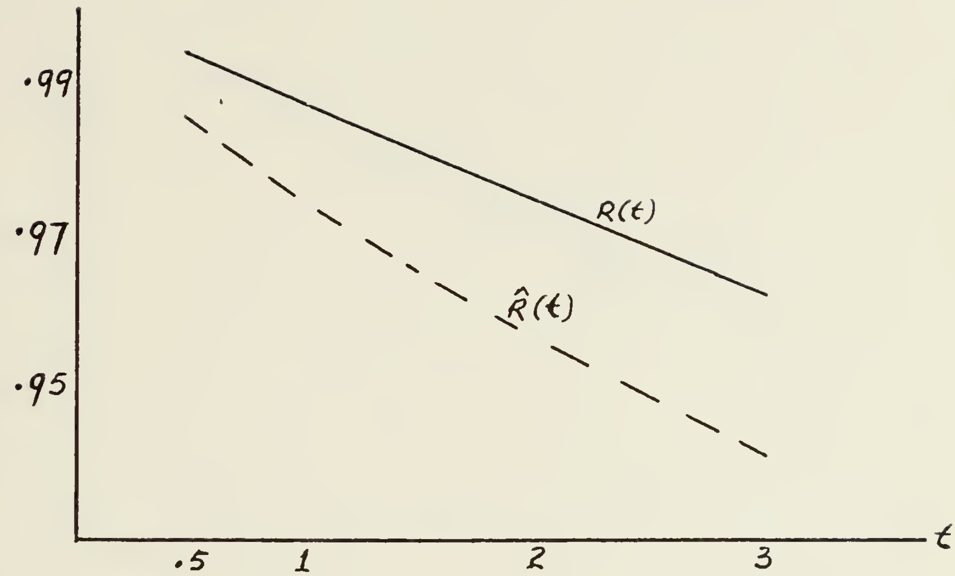

Figure 29

N = 5

T_o = 20

r = 50

λ:	.005	.015	.010	.015	.020
p:	.2	.2	.2	.2	.2

k:	0	1	2	3
PB:	0	5	4	1
NB:	13	15	9	3
Total:	13	20	13	4

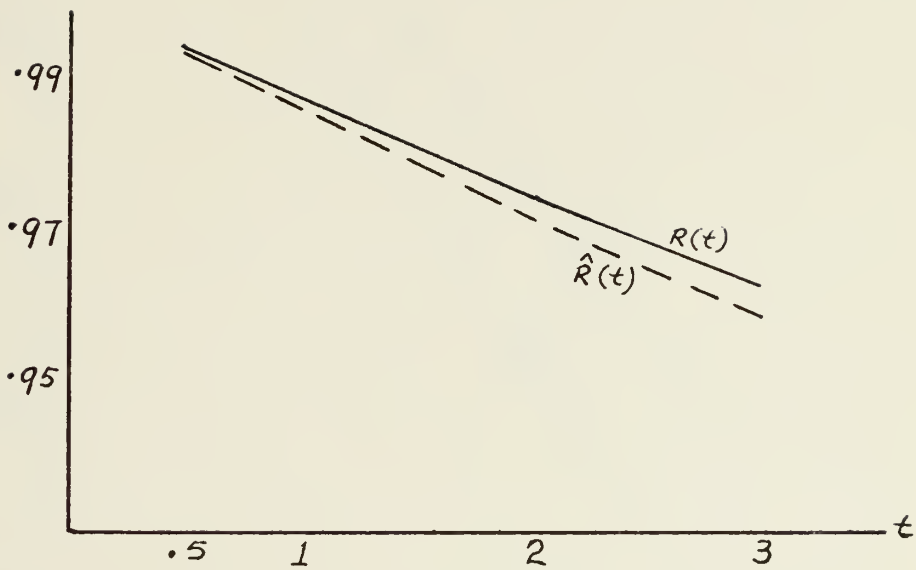

Figure 30

N = 50
T_o = 20
r = 50

λ:	.005	.015	.010	.015	.020
p:	.2	.2	.2	.2	.2

k:	4	5	6	7	8	9	10	11	12	13	14	15	16	17 – 21
PB:	0	0	1	1	0	1	2	0	1	0	1	0	1	0 0
NB:	1	0	1	2	3	2	6	6	7	3	5	1	3	1 1
Total:	1	0	2	3	3	3	8	6	8	3	6	1	4	1 1

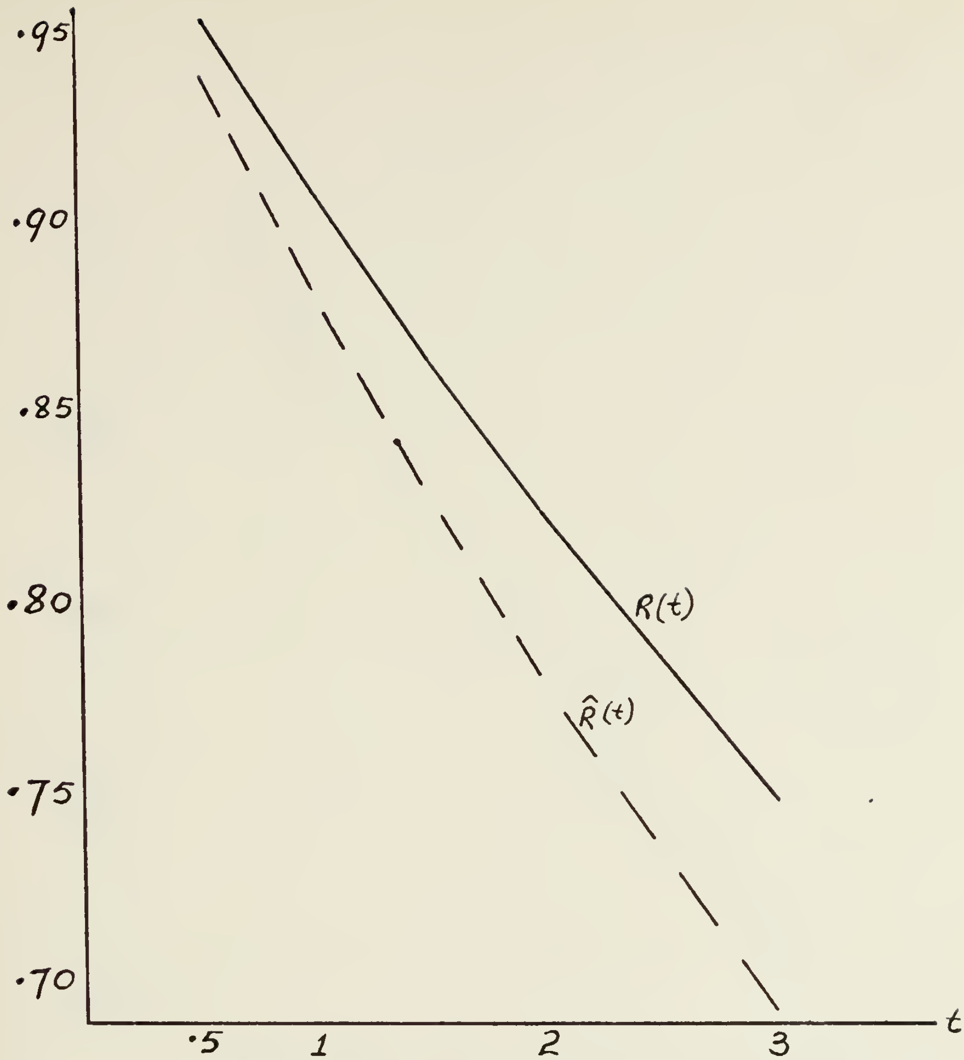

Figure 31

N = 5

T_o = 20

r = 50

λ:	.01	.05	.08	.10	.15
p:	.05	.10	.20	.40	.25

k:	2	3	4	5
PB:	3	1	0	0
NB:	0	4	19	23
Total:	3	5	19	23

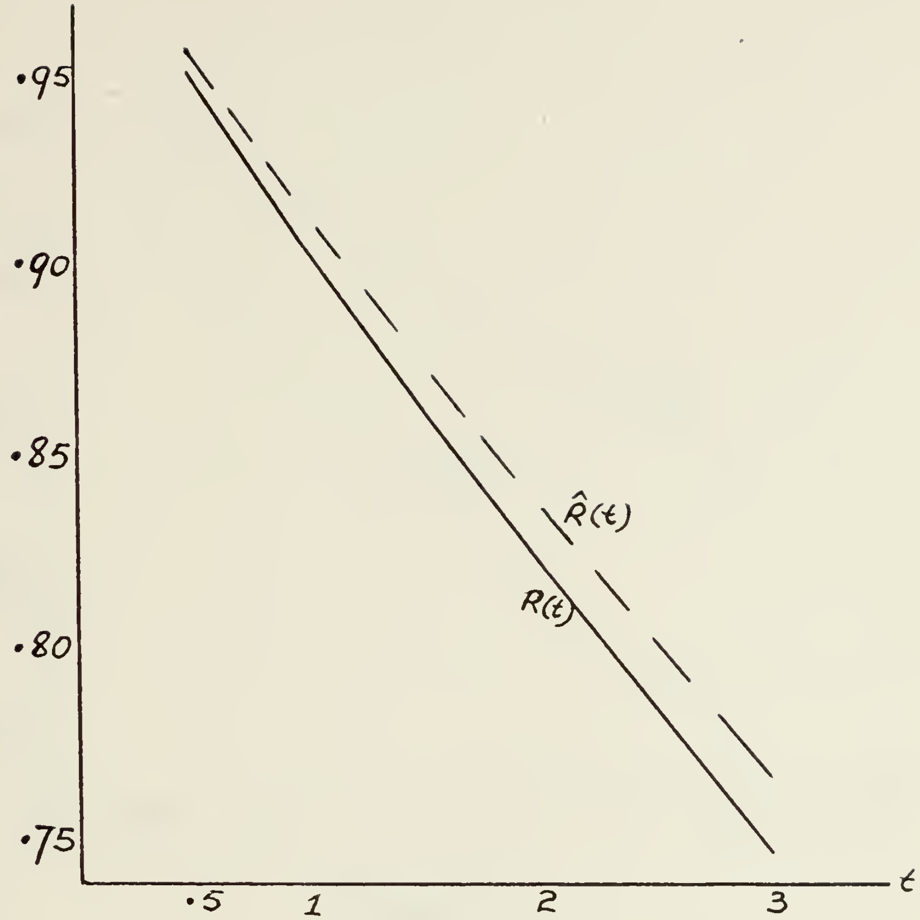

Figure 32

$N = 50$
$T_o = 20$
$r = 50$

λ:	.01	.05	.08	.10	.15
p:	.05	.10	.20	.40	.25

k:	35	36	37	38	39	40	41	42	43	44	45
PB:	0	0	0	0	0	0	0	0	0	0	0
NB:	1	1	4	4	4	10	7	10	3	3	3
Total:	1	1	4	4	4	10	7	10	3	3	3

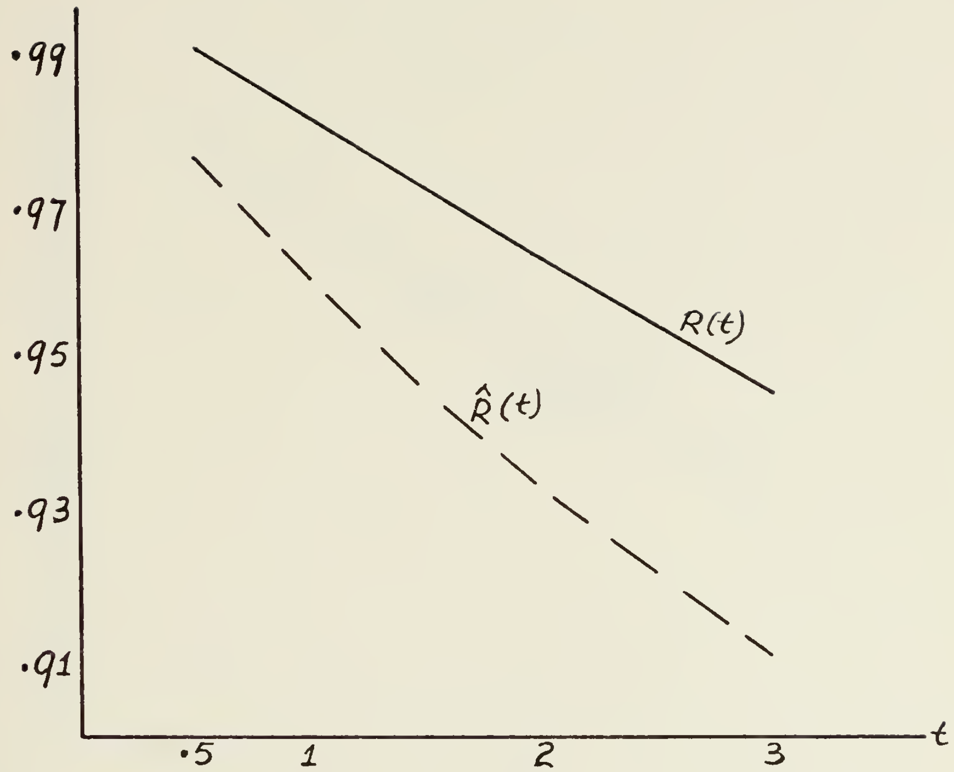

Figure 33

N = 5

T_o = 20

r = 50

λ:	.005	.007	.010	.015	.017	.020	.025	.027	.030	.035
p:	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1

k:	0	1	2	3
PB:	0	6	7	2
NB:	7	9	14	5
Total:	7	15	21	7

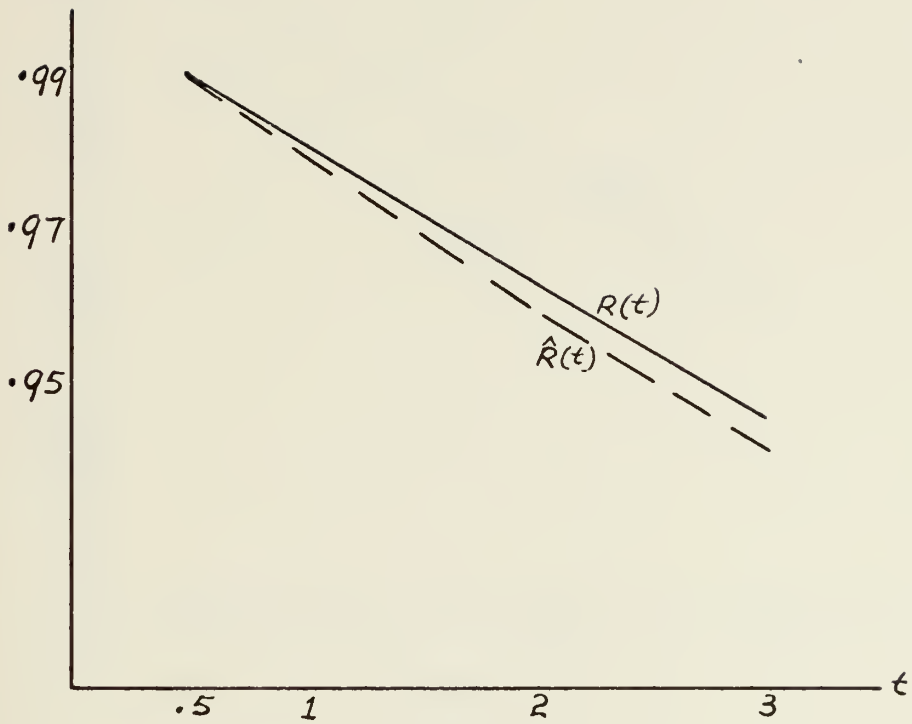

Figure 34

N = 50

T_o = 20

r = 50

λ:	.005	.007	.010	.015	.017	.020	.025	.027	.030	.035
p:	.1	.1	.1	.1	.1	.1	.1	.1	.1	.1

k:	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
PB:	0	0	0	0	1	2	2	0	1	0	0	0	0	0	0
NB:	1	3	2	2	4	8	4	5	1	4	2	4	1	2	1
Total:	1	3	2	2	5	10	6	5	2	4	2	4	1	2	1

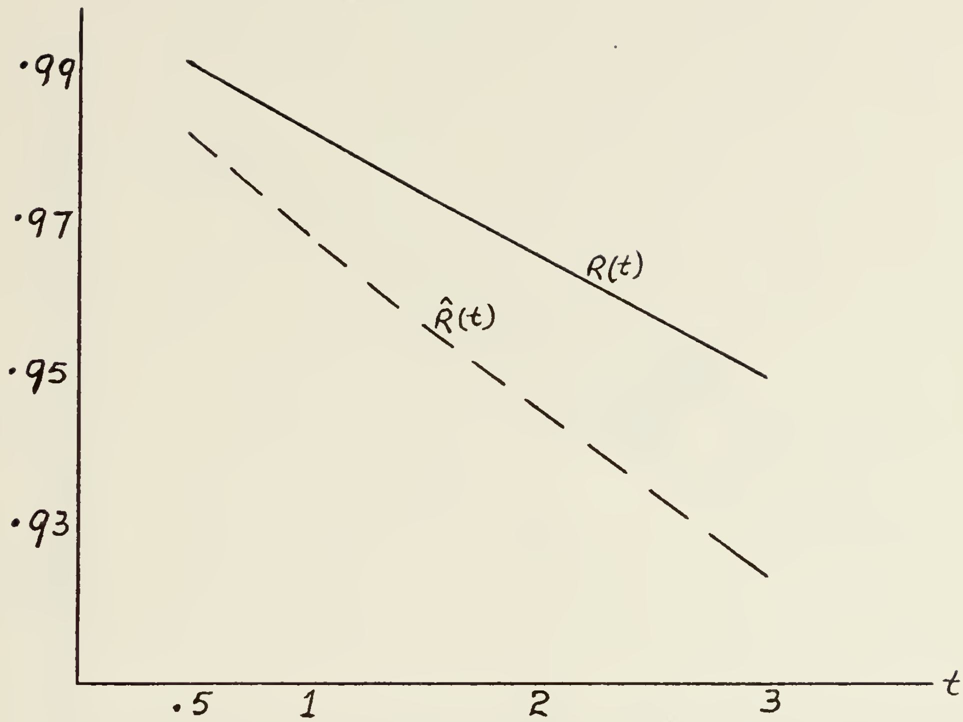

Figure 35

N = 5

T_o = 20

r = 50

λ:	.005	.007	.010	.015	.017	.020	.025	.027	.030	.035
p:	.03	.05	.10	.20	.25	.20	.07	.05	.03	.02

k:	0	1	2	3	4
PB:	0	7	7	2	0
NB:	13	11	5	4	1
Total:	13	18	12	6	1

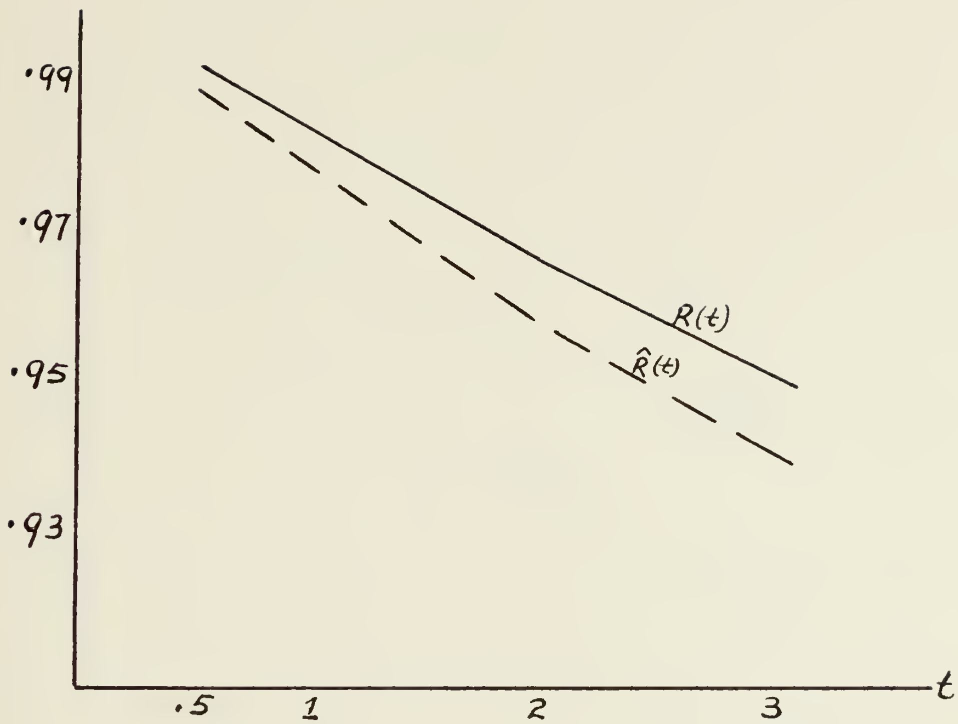

Figure 36

N = 50

T_o = 20

r = 50

λ:	.005	.007	.010	.015	.017	.020	.025	.027	.030	.035
p:	.03	.05	.10	.20	.25	.20	.07	.05	.03	.02

k:	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
PB:	1	1	0	0	1	0	3	1	1	0	1	1	1	1	0	0	0	0	0
NB:	0	0	0	1	1	4	3	3	3	3	7	4	3	4	1	0	0	0	1
Total:	1	1	0	1	2	4	6	4	4	3	8	5	4	5	1	0	0	0	1

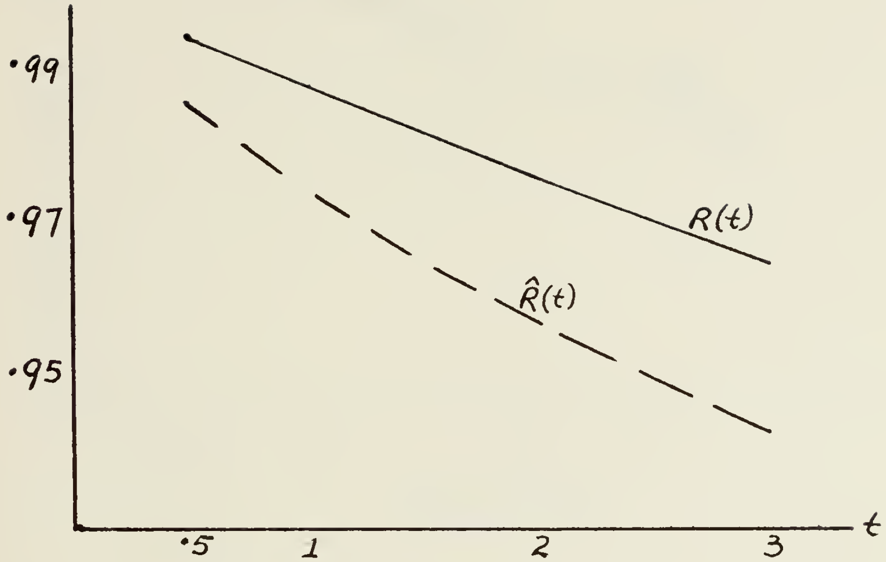

Figure 37

$N = 5$

$T_o = 20$

$r = 50$

λ:	.005	.007	.010	.015	.017	.020	.025	.027	.030	.035
p:	.25	.20	.20	.10	.07	.05	.05	.03	.03	.02

k:	0	1	2	3
PB:	0	12	2	1
NB:	15	11	6	3
Total:	15	23	8	4

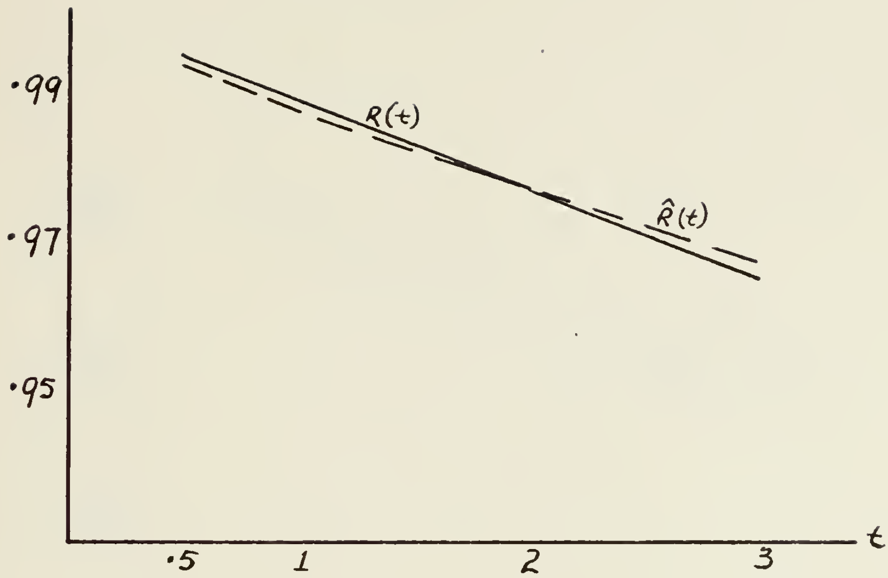

Figure 38

N = 10

T_o = 20

r = 50

λ:	.005	.007	.010	.015	.017	.020	.025	.027	.030	.035
p:	.25	.20	.20	.010	.007	.005	.005	.003	.003	.002

k:	0	1	2	3	4	5
PB:	0	5	3	7	0	0
NB:	2	7	15	6	3	2
Total:	2	12	18	13	3	2

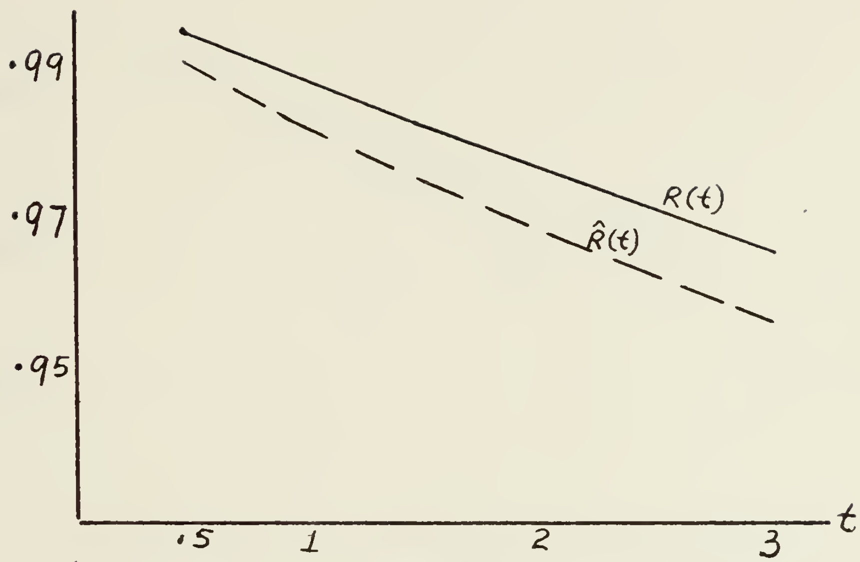

Figure 39

N = 20

T_o = 20

r = 50

λ:	.005	.007	.010	.015	.017	.020	.025	.027	.030	.035
p:	.25	.20	.20	.10	.07	.05	.05	.03	.03	.02

k:	0	1	2	3	4	5	6	7	8
PB:	0	2	1	2	4	1	2	1	1
NB:	1	3	5	6	6	10	2	2	1
Total:	1	5	6	8	10	11	4	3	2

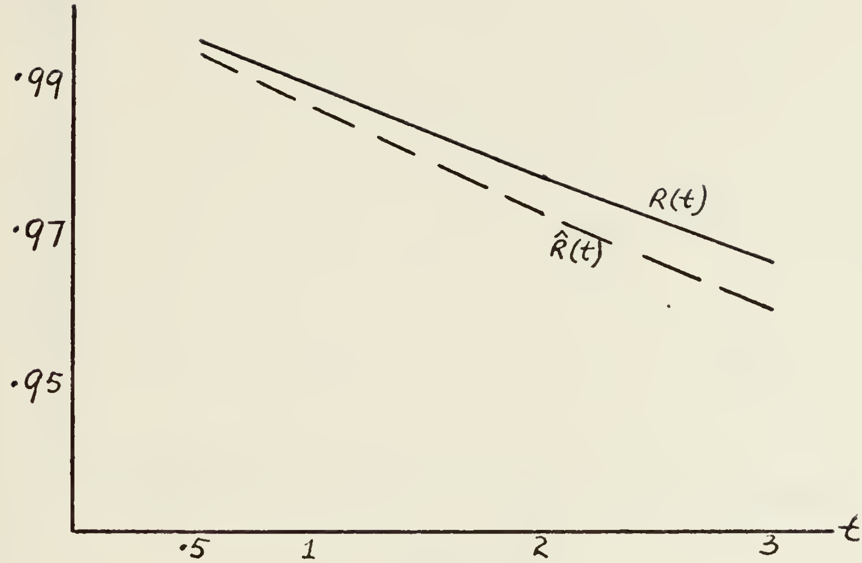

Figure 40

N = 50

T_o = 20

r = 50

λ:	.005	.007	.010	.015	.017	.020	.025	.027	.030	.035
p:	.25	.20	.20	.10	.07	.05	.05	.03	.03	.02

k:	4	5	6	7	8	9	10	11	12	13	14	15
PB:	1	0	1	1	0	3	1	1	0	0	1	1
NB:	1	0	1	4	4	2	7	3	6	5	6	1
Total:	1	0	1	5	4	5	8	4	6	5	7	2

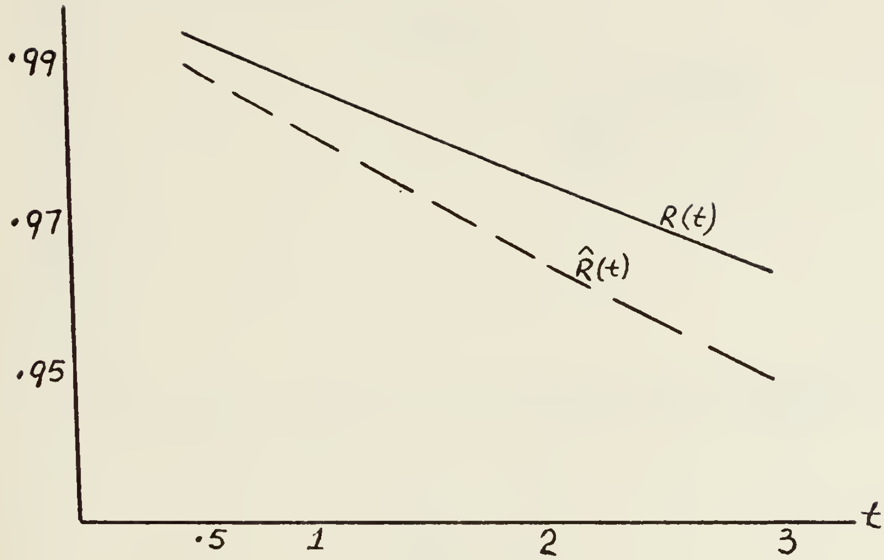

Figure 41

N = 10
r = 50

λ:	.005	.02
p:	.50	.50

k:	3
PB:	4
NB:	46
Total:	50

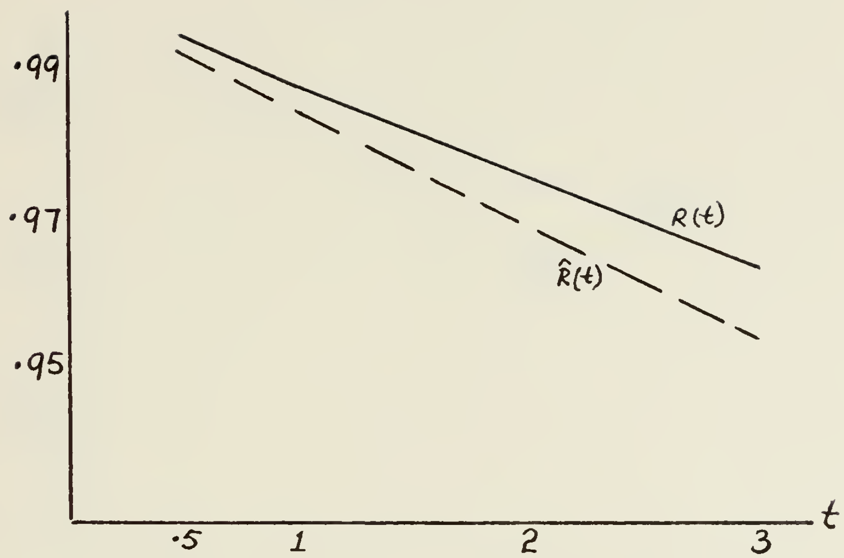

Figure 42

N = 10

r = 50

λ:	.005	.020
p:	.50	.50

k:	5
PB:	8
NB:	42
Total:	50

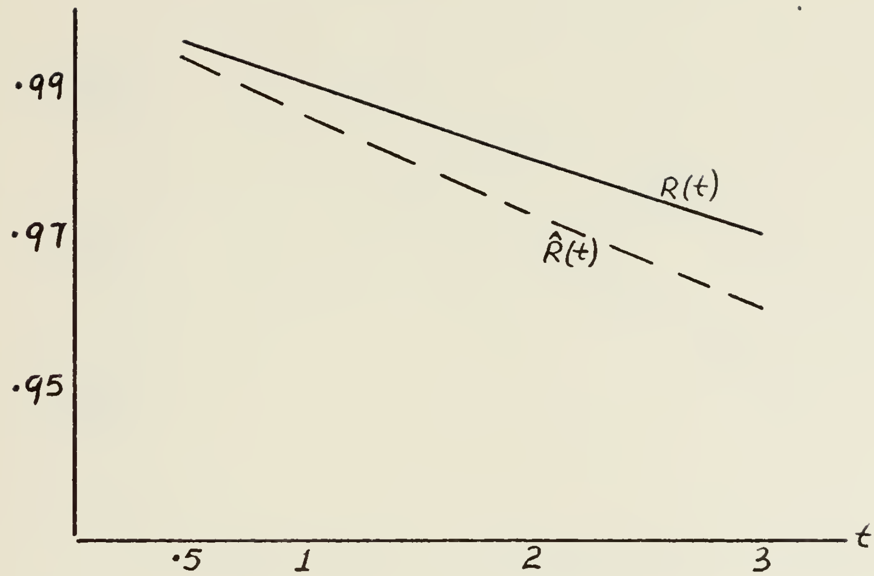

Figure 43

N = 10
r = 50

λ:	.005	.010	.015	.020
p:	.40	.20	.35	.05

k:	3
PB:	4
NB:	46
Total:	50

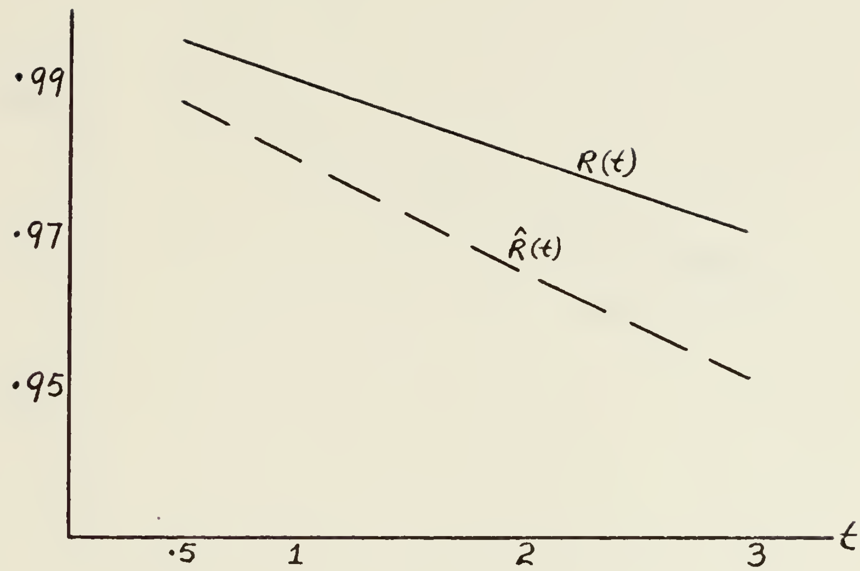

Figure 44

N = 10
r = 50

λ:	.005	.010	.015	.020
p:	.40	.20	.35	.05

k:	5
PB:	13
NB:	37
Total:	50

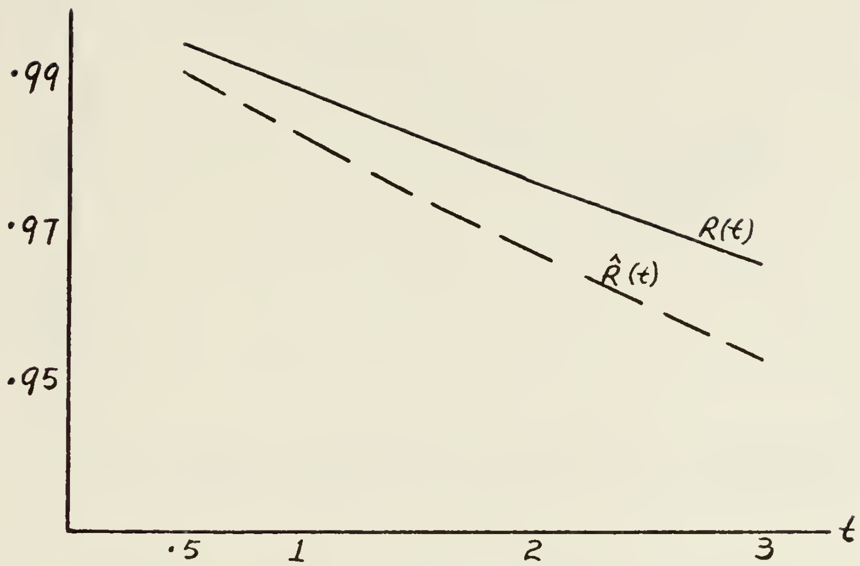

Figure 45

N = 10

r = 50

λ:	.005	.007	.010	.015	.017	.020	.025	.027	.030	.035
p:	.25	.20	.20	.10	.07	.05	.05	.03	.03	.02

k: 3

PB: 4

NB: 46

Total: 50

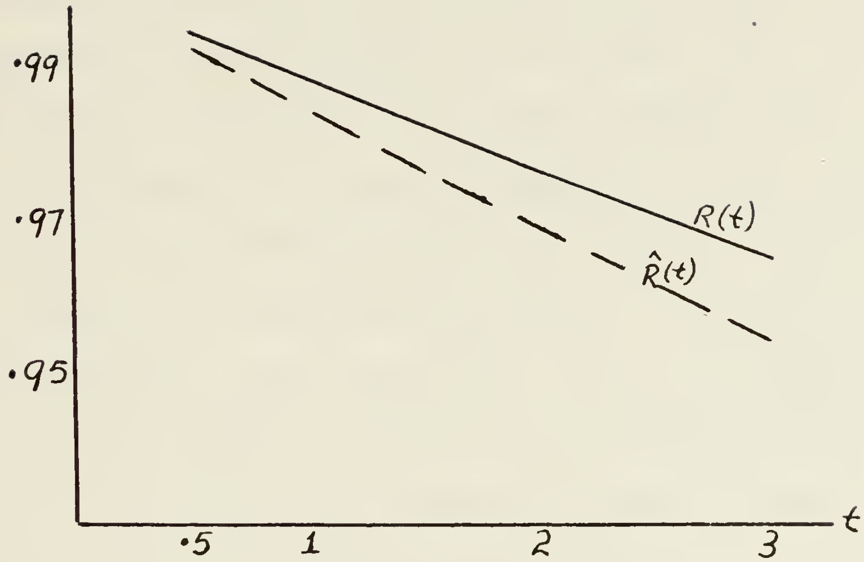

Figure 46

N = 10

r = 50

λ:	.005	.007	.010	.015	.017	.020	.025	.027	.030	.035
p:	.25	.20	.20	.10	.07	.05	.05	.03	.03	.02

k = 5

PB = 9

NB = 41

Total = 50

V. CONCLUSION

The results of the simulations show that

1. In all cases, under the first procedure of simulation a positive solution for $\hat{\beta}$ was never obtained for relatively small planned test times, e.g., $T_o = 10$. Some positive solutions for $\hat{\beta}$ were obtained when $T_o = 20$.

2. By using the second procedure (fixed number of failures) a positive solution for $\hat{\beta}$ never existed for $k = 1$ and $k = 2$. Only for $k \geq 3$ were positive solutions for $\hat{\beta}$ obtained. The larger the value of k, the more accurate the computed reliability estimate will be.

3. The result also showed that most of the reliability estimates have quite small error relative to the true value of the reliability. It was less than 1% on the average. The smallest relative error of the reliability estimates was .02% while the largest one was about 3.15%. Thus the proposed estimation procedure is quite accurate when the underlying distribution on Λ is a discrete distribution similar to those assumed rather than a continuous gamma distribution. In this regard the estimation procedure indicates some property of robustness.

SYMBOLS USED IN THE COMPUTER OUTPUTS

N: Number of generated component's life times

T_o: Planned test time

Parameter: Parameters (λ) of the life times

Probability: Discrete probability density of λ

k: Number of failure components on test

PB: Number of replications that gave a solution for $\beta > 0$, associated with k

NB: Number of replications that did not give a solution for $\beta > 0$, associated with k

Total: Number of replications that gave k failures

T: Time to compute the reliability and its estimate

R(T): True value of the reliability

RE(T): Reliability estimate of R(T)

SD: Standard deviation of 50 replications of RE(T)

MOE: Measure of Effectiveness of estimation, i.e., RE(T) - R(T)

COMPUTER OUTPUT

```
C
C     COMPUTATION RELIABILITY OF COMPONENTS FROM A SAMPLE
C     CONTAINING COMPONENTS WITH MIXED FAILURE RATES.
C     FAILURE RATE TO BE ASSUMED AS GAMMA DISTRIBUTED RANDOM
C     VARIABLE WITH PARAMETERS ALPHA AND BETA.
C
      DIMENSION R(5),T(50),X(50),PARM(10),P(10),PR(10),
     1Y(50),DEV(5),RTEST(50,5),SD(5),KEY(50),NK(50),
     2RT(5),RTA(5),NFAIL(50),NOBETA(50),KH(50)
      DATA N,NG,NP,NR/10,50,10,5/
      CALL OVFLOW
      IX=11121941
      XNG=NG
      XN=N
      TO=20.
      DO 222 NCASE=1,3
      READ(5,2010)(R(I),I=1,NR)
 2010 FORMAT(5F10.2)
      READ(5,2011)(PARM(I),I=1,NP)
      READ(5,2011)(P(I),I=1,NP)
 2011 FORMAT(10F5.3)
      DO 203 I=1,NR
      RT(I)=0.
      DO 203 J=1,NP
  203 RT(I)=RT(I)+P(J)*EXP(-PARM(J)*R(I))
C
C     GENERATE RANDOM VARIABLE T ( EXPONENTIAL DISTRIBUTED )
C
      TEMP=0.
      DO 204 I=1,NP
      PR(I)=P(I)+TEMP
  204 TEMP=PR(I)
      NFO=0
      DO 210 I=1,NG
      NK(I)=0
      NOBETA(I)=0
  210 NFAIL(I)=0
      DO 212 JN=1,NG
   15 CALL SEXPON(IX,Y,N)
      CALL RANDOM(IX,X,N)
      DO 211 I=1,N
  211 T(I)=0.
      DO 208 I=1,N
      IF(Y(I).LT.1.E-10)Y(I)=0.
      IF(Y(I).GT.1.E06)Y(I)=1.E06
      DO 213 J=1,NP
      IF(X(I).GT.PR(J))GO TO 213
      T(I)=Y(I)/PARM(J)
```

```
      GO TO 208
  213 CONTINUE
  208 CONTINUE
      CALL SHSORT(T,KEY,N)
      IF(T(1).LE.TO)GO TO 214
      NFO=NFO+1
      XK=0
      GO TO 24
  214 DO 216 I=1,N
      IF(T(I).GT.TO)GO TO 215
      K=I
      GO TO 216
  215 T(I)=TO
  216 CONTINUE
      XK=K
      NFAIL(K)=NFAIL(K)+1
C
C     CHECK IF F(BETA=SMALL) TO BE POSITIVE
C
      B=.0001
      CALL FDBETA(T,N,K,B,FB,DFB)
      IF (FB.GT.0.) GO TO 17
      IF(FB.LT.0.)GO TO 16
      BC=B
      GO TO 43
   16 NOBETA(K)=NOBETA(K)+1
      GO TO 18
C
C     COMPUTATION BETA AND ALPHA
C
   17 BI=2.
      B=BI
  234 CALL FDBETA(T,N,K,BI,FB,DFB)
      IF (ABS(FB).GT..0001) GO TO 56
      BC=BI
      GO TO 43
   56 IF(FB.LT.0.)GO TO 22
      IF(DFB.GT.-.2)GO TO 23
   22 BC=BI-FB/DFB
      IF (BC.LE.0.) GO TO 65
      IF(BC.GT.5000.)GO TO 18
C
C     (BETA.LE.5000). COMPUTE BETA.
C
      IF(BC.GE.500.)GO TO 500
      IF(ABS(BC-BI).LE..001)GO TO 43
      GO TO 600
  500 IF(ABS(BC-BI).LE..1)GO TO 43
```

```
600 BI=.5*(BC+BI)
    GO TO 234
 65 BI=.5*BI
    GO TO 234
 23 BI=2.*BI
    GO TO 234
 43 BETA=BC
    YY=0.
    DO 44  I=1,N
 44 YY=YY+ALOG(1.+T(I)*BETA)
    ALPHA=XK/YY
    GAMMA=ALPHA*BETA
    NK(K)=NK(K)+1
    DO 501  I=1,NR
501 RTEST(JN,I)=1./((1.+R(I)*BETA)**ALPHA)
    GO TO 212
 18 ST=0.
    DO 220  I=1,K
220 ST=ST+T(I)
 24 ESLADA=XK/(ST+(XN-XK)*TO)
    DO 19  I=1,NR
 19 RTEST(JN,I)=EXP(-ESLADA*R(I))
212 CONTINUE
    DO 502  I=1,NR
    RTA(I)=0.
    DO 503  JN=1,NG
503 RTA(I)=RTA(I)+RTEST(JN,I)/XNG
    SS=0.
    DO 504  JN=1,NG
504 SS=SS+(RTEST(JN,I)-RTA(I))**2
    SD(I)=SQRT(SS/(XNG-1.))
502 DEV(I)=RTA(I)-RT(I)
255 DO 252  I=1,NG
252 KH(I)=I
    WRITE(6,253)
253 FORMAT('1'/////////////////////)
    WRITE(6,218)N,TO,(PARM(I),I=1,NP),(P(I),I=1,NP)
218 FORMAT(20X,'N   =',I3/20X,'TO =',F4.0//20X,'PARAMETER',
   13X,':',10F6.3/20X,'PROBABILITY :',10F6.3///)
    WRITE(6,219)(KH(I),I=1,N),(NK(I),I=1,N),NFO,(NOBETA(I)
   1,I=1,N),NFO,(NFAIL(I),I=1,N)
219 FORMAT(20X,'K     :    0',10I3//20X,'PB    :    0',10I3/
   120X,'NB    : ',11I3/20X,'TOTAL: ',11I3///)
    WRITE(6,228)(R(I),I=1,NR),(RT(I),I=1,NR),(RTA(I),I=1,
   1NR),(SD(I),I=1,NR),(DEV(I),I=1,NR)
228 FORMAT(20X,'T     :',5(F7.1,1X)//20X,'R(T) :',5F8.4/20X
   1,'RE(T):',5F8.4/20X,'SD    :',5F8.4/20X,'MOE   :',5F8.4)
222 CONTINUE
```

```
      WRITE(6,229)
  229 FORMAT('1','    ')
      STOP
      END
C
C     SUBROUTINE FDBETA(T,N,K,B,FB,DFB) TO COMPUTE THE VALUE
C     OF FB=F(BETA) AND DFB=DERIVATIVE OF F(BETA), WHERE
C     T=GENERATED FAILURE TIME, N=NUMBER OF GENERATED T'S,
C     K=NUMBER OF FAILURES, AND B=VALUE OF BETA.
C
      SUBROUTINE FDBETA(T,N,K,B,FB,DFB)
      DIMENSION T(30)
      X=0.
      Y=0.
      Z=0.
      DX=0.
      DZ=0.
      XK=K
      DO 320 I=1,N
      TB=1.+T(I)*B
      IF(I.GT.K)GO TO 310
      X=X+1./TB
      DX=DX-T(I)/TB**2
  310 Y=Y+ALOG(TB)
      Z=Z+T(I)/TB
      DY=Z
  320 DZ=DZ-(T(I)/TB)**2
      FB=(1./XK)*X*Y/Z-B
      DFB=(1./XK)*(DX*Y*Z+X*DY*Z-X*Y*DZ)/Z**2-1.
      RETURN
      END
```

N = 10
TO = 20.

PARAMETER : 0.010 0.010
PROBABILITY : 0.500 0.500

K :	0	1	2	3	4	5	6	7	8	9	10
PB :	0	9	5	3	0	1	0	0	0	0	0
NB :	7	8	8	5	3	1	0	0	0	0	0
TOTAL:	7	17	13	8	3	2	0	0	0	0	0

T :	0.5	1.0	1.5	2.0	3.0
R(T) :	0.9950	0.9900	0.9851	0.9802	0.9704
RE(T):	0.9865	0.9780	0.9707	0.9642	0.9524
SD :	0.0263	0.0347	0.0407	0.0454	0.0529
MOE :	-0.0085	-0.0121	-0.0144	-0.0160	-0.0180

N = 10
TO = 20.

PARAMETER : 0.010 0.020
PROBABILITY : 0.500 0.500

K	0	1	2	3	4	5	6	7	8	9	10
PB	0	6	5	7	2	0	0	0	0	0	0
NB	4	2	7	8	4	5	0	0	0	0	0
TOTAL	4	8	12	15	6	5	0	0	0	0	0

T	0.5	1.0	1.5	2.0	3.0
R(T)	0.9925	0.9851	0.9778	0.9705	0.9561
RE(T)	0.9835	0.9717	0.9614	0.9520	0.9348
SD	0.0222	0.0302	0.0358	0.0403	0.0479
MOE	-0.0090	-0.0134	-0.0164	-0.0185	-0.0213

N = 10
TO = 20.

PARAMETER : 0.005 0.020
PROBABILITY : 0.500 0.500

K :	0	1	2	3	4	5	6	7	8	9	10
PB :	0	5	6	3	2	0	0	0	0	0	0
NB :	5	7	9	7	3	3	0	0	0	0	0
TOTAL:	5	12	15	10	5	3	0	0	0	0	0

T :	0.5	1.0	1.5	2.0	3.0
R(T) :	0.9938	0.9876	0.9815	0.9754	0.9634
RE(T):	0.9851	0.9749	0.9663	0.9586	0.9448
SD :	0.0202	0.0297	0.0358	0.0405	0.0478
MOE :	-0.0087	-0.0127	-0.0152	-0.0168	-0.0186

N = 10
TO = 20.

PARAMETER :	0.005	0.015	0.010	0.015	0.020
PROBABILITY :	0.200	0.200	0.200	0.200	0.200

K :	0	1	2	3	4	5	6	7	8	9	10
PB :	0	3	5	5	2	0	0	0	0	0	0
NB :	6	4	10	7	7	1	0	0	0	0	0
TOTAL:	6	7	15	12	9	1	0	0	0	0	0

T :	0.5	1.0	1.5	2.0	3.0
R(T) :	0.9935	0.9871	0.9807	0.9744	0.9619
RE(T):	0.9873	0.9775	0.9689	0.9609	0.9463
SD :	0.0205	0.0294	0.0354	0.0400	0.0473
MOE :	-0.0062	-0.0096	-0.0118	-0.0135	-0.0156

N = 10
TO = 20.

PARAMETER	: 0.050	0.060	0.040	0.020	0.010
PROBABILITY	: 0.400	0.250	0.200	0.100	0.050

K :	0	1	2	3	4	5	6	7	8	9	10
PB :	0	0	0	1	4	2	2	0	0	0	0
NB :	0	0	1	0	4	10	15	6	4	1	0
TOTAL:	0	0	1	1	8	12	17	6	4	1	0

T :	0.5	1.0	1.5	2.0	3.0
R(T) :	0.9775	0.9556	0.9342	0.9134	0.8732
RE(T):	0.9664	0.9403	0.9168	0.8948	0.8543
SD :	0.0359	0.0478	0.0549	0.0600	0.0676
MOE :	-0.0112	-0.0153	-0.0175	-0.0185	-0.0188

N = 10
TO = 20.

PARAMETER :	0.100	0.150	0.080	0.050	0.010
PROBABILITY :	0.400	0.250	0.200	0.100	0.050

K :	0	1	2	3	4	5	6	7	8	9	10
PB :	0	0	0	0	0	0	1	3	0	0	0
NB :	0	0	0	0	1	0	3	5	16	15	6
TOTAL:	0	0	0	0	1	0	4	8	16	15	6

T :	0.5	1.0	1.5	2.0	3.0
R(T) :	0.9519	0.9064	0.8633	0.8226	0.7477
RE(T):	0.9477	0.9000	0.8559	0.8148	0.7402
SD :	0.0240	0.0413	0.0550	0.0663	0.0842
MOE :	-0.0042	-0.0064	-0.0074	-0.0078	-0.0075

N = 10
TO = 20.

PARAMETER :	0.005	0.007	0.010	0.015	0.017	0.020	0.025	0.027	0.030	0.035
PROBABILITY :	0.100	0.100	0.100	0.100	0.100	0.100	0.100	0.100	C.100	0.100

K :	0	1	2	3	4	5	6	7	8	9	10
PB :	0	1	6	6	2	0	1	0	0	0	0
NB :	3	2	7	7	6	9	0	0	0	0	0
TOTAL:	3	3	13	13	8	9	1	0	0	0	0

T :	0.5	1.0	1.5	2.0	3.0
R(T) :	0.9905	0.9811	0.9719	0.9627	0.9447
RE(T):	0.9831	0.9697	0.9577	0.9466	0.9261
SD :	0.0259	0.0375	0.0449	0.0505	0.0591
MOE :	-0.0074	-0.0114	-0.0142	-0.0161	-0.0186

N = 10
TO = 20.

PARAMETER :	0.017	0.015	0.020	0.010	0.025	0.007	0.027	0.005	0.030	0.035
PROBABILITY :	0.250	0.200	0.200	0.100	0.070	0.050	0.050	0.030	0.030	0.020

K :	0	1	2	3	4	5	6	7	8	9	10
PB :	0	4	6	5	1	0	0	0	0	0	0
NB :	1	6	4	11	6	4	2	0	0	0	0
TOTAL:	1	10	10	16	7	4	2	0	0	0	0

T :	0.5	1.0	1.5	2.0	3.0
R(T) :	0.9913	0.9827	0.9742	0.9658	0.9492
RE(T):	0.9856	0.9740	0.9637	0.9540	0.9360
SD :	0.0133	0.0202	0.0252	0.0294	0.0370
MOE :	-0.0057	-0.0087	-0.0105	-0.0118	-0.0132

N = 10
TO = 20.

PARAMETER :	0.005	0.007	0.010	0.015	0.017	0.020	0.025	0.027	0.030	0.035
PROBABILITY :	0.250	0.200	0.200	0.100	0.070	0.050	0.050	0.030	0.030	0.020

K :	0	1	2	3	4	5	6	7	8	9	10
PB :	0	6	7	4	2	0	0	0	0	0	0
NB :	6	5	12	3	5	0	0	0	0	0	0
TOTAL:	6	11	19	7	7	0	0	0	0	0	0

T :	0.5	1.0	1.5	2.0	3.0
R(T) :	0.9940	0.9881	0.9822	0.9764	0.9649
RE(T):	0.9861	0.9760	0.9675	0.9599	0.9465
SD :	0.0182	0.0278	0.0346	0.0400	0.0482
MOE :	-0.0080	-0.0121	-0.0147	-0.0165	-0.0184

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S8637 Sunjata
c.1 Sensitivity analysis of a reliability estimation procedure for a component whose failure density is a mixture of exponential failure densities.